A. HALD AND P. THYREGOD

BAYESIAN SINGLE SAMPLING PLANS BASED ON LINEAR COSTS AND THE POISSON DISTRIBUTION

Det Kongelige Danske Videnskabernes Selskab
Matematisk-fysiske Skrifter 3, 7

Kommissionær: Munksgaard
København 1971

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Selskabets sekretariat og postadresse: Dantes Plads 5, 1556 København V.

The address of the secretariate of the Academy is:

Det Kongelige Danske Videnskabernes Selskab,
Dantes Plads 5, 1556 København V, Denmark.

Selskabets kommissionær: Munksgaard's Forlag, Prags Boulevard 47, 2300 København S.

The publications are sold by the agent of the Academy:

Munksgaard, *Publishers,*
47 Prags Boulevard, 2300 København S, Denmark.

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Synopsis

Suppose that the output of a controlled process with an average of λ defects per unit is divided into inspection lots of M units and that a random sample of m units is inspected from each lot. A lot is accepted if the total number of defects found in the sample is less than or equal to c, otherwise the lot is rejected. The average probability of acceptance then becomes

$$B(c, m\lambda) = \sum_{x=0}^{c} e^{-m\lambda}(m\lambda)^x / x!$$

for $c = 0, 1, \ldots, \lambda \geqq 0$ and $m \geqq 0$. We assume that the costs of inspection, acceptance and rejection per unit are linear functions of λ and that the observational unit has been chosen such that the break-even quality equals 1, i.e. a process with $\lambda \leqq 1$ should be accepted whereas for $\lambda > 1$ it should be rejected. Finally it is assumed that λ is a random variable with non-degenerate distribution function $W(\lambda)$, the prior distribution.

The expected regret for a lot of M units may then be written in standardized form as

$$R(c,m,M) = m\delta + (M - m)\, d(c,m) \quad \text{for} \quad 0 \leqq m \leqq M,$$

where $d(c,m)$ represents the expected decision loss per unit, i.e.

$$d(c,m) = \int_0^1 (1 - \lambda)\{1 - B(c, m\lambda)\}\, dW(\lambda) + \int_1^\infty (\lambda - 1)\, B(c, m\lambda)\, dW(\lambda),$$

and δ is a positive constant related to the sampling and inspection costs per unit. The Bayesian single sampling plan is defined as the plan minimizing $R(c,m,M)$ with respect to (c,m) taking into account that min $R = R(M)$ should be compared with the costs of accepting (rejecting) without inspection.

In the first part of the paper the problem of determining the Bayesian plan is discussed for an arbitrary prior distribution. The properties of $d(c,m)$ and $R(c,m,M)$ are derived and it is shown how the Bayesian plan may be constructed. It is proved that $R(M)$, the minimum regret, is an increasing, continuous and piecewise differentiable function of M with decreasing slope, and that the optimum acceptance number and the optimum sample size are increasing functions of M.

In the second part of the paper the Bayesian plans are discussed for two important special cases, viz. for the two-point prior distribution and the gamma prior distribution. The exact solution is supplemented by an approximate solution derived from asymptotic expansions for $M \to \infty$. Finally it is shown that the Poisson solution may be used as a good approximation to the binomial solution under certain reasonable conditions.

For the two-point prior distribution the present paper supplements the results of a previous paper regarding the Bayesian plan in the binomial case, see Mat. Fys. Skr. Dan. Vid. Selsk. 3, No. 2, 1965.

The Appendix contains tables of Bayesian plans for the two types of prior distributions.

PRINTED IN DENMARK
BIANCO LUNOS BOGTRYKKERI A/S

CONTENTS

1. Introduction and summary

Consider a problem with a choice between two decisions, acceptance and rejection, say, based on the observation of a Poisson distributed variable. Let λ denote the mean occurrence rate per observational unit, i.e. λ may denote the expected number of defects per unit by inspection of an industrial product. Suppose that the output of a controlled process with an average of λ defects per unit is divided into inspection lots of M units and that a random sample of m units is inspected from each lot. A lot is accepted if the total number of defects found in the sample is less than or equal to c, the acceptance number, otherwise the lot is rejected. The average probability of acceptance then becomes

$$B(c,m\lambda) = \sum_{x=0}^{c} b(x,m\lambda) = \sum_{x=0}^{c} e^{-m\lambda}(m\lambda)^x/x!$$

for $c = 0,1,\ldots, \lambda \geqq 0$ and $m \geqq 0$. To shorten the notation we also write $P(\lambda) = B(c,m\lambda)$ and $Q(\lambda) = 1 - P(\lambda)$.

We assume that the costs of inspection, acceptance and rejection per unit are linear functions of λ and, if the contrary is not explicitly stated, that the observational unit has been chosen such that the break-even quality equals 1, i.e. a process with $\lambda \leqq 1$ should be accepted whereas for $\lambda > 1$ it should be rejected.

Finally it is assumed that λ is a random variable with non-degenerate distribution function $W(\lambda)$, the prior distribution.

The expected regret for a lot of M units may then be written in standardized form as

$$R(c,m,M) = m\delta + (M-m)\,d(c,m), \qquad 0 \leqq m \leqq M, \tag{1}$$

where $d(c,m)$ represents the expected decision loss per unit, i.e.

$$d(c,m) = \int_0^1 (1-\lambda)\{1-B(c,m\lambda)\}\,dW(\lambda) + \int_1^\infty (\lambda-1)\,B(c,m\lambda)\,dW(\lambda), \tag{2}$$

and δ is a positive constant related to the sampling and inspection costs per unit.

The optimum or Bayesian single sampling plan is defined as the plan minimizing $R(c,m,M)$ with respect to (c,m), taking into account that $\min R = R(M)$, say, should be compared with the costs of accepting (rejecting) without inspection. Sampling should be employed only if it leads to the cheaper solution.

In the paper the basic properties of the optimum plan are derived and a method for determining such plans is given. In the Appendix optimum plans are tabulated for two families of prior distributions, the two-point distribution and the gamma distribution. A general procedure for tabulating the optimum plans has been developed as a FORTRAN program and may be obtained from the authors.

The Poisson solution gives an approximation to the corresponding binomial solution defined by minimizing the regret function

$$R_b(c,n,N) = n\delta_1 + (N-n)\left\{\int_0^{p_r}(p_r - p)Q(p)dF(p) + \int_{p_r}^{1}(p-p_r)P(p)dF(p)\right\}, \qquad (3)$$

where N denotes lot size, n sample size, c the acceptance number, p_r the break-even fraction defective and $P(p) = B(c,n,p)$ the cumulative binomial distribution.

Writting $M = Np_r$, $m = np_r$, $\lambda = p/p_r$ and $B(c,n,p) \sim B(c,np) = B(c,m\lambda)$ it follows that $R(c,m,M)$ may be obtained as the limit of $R_b(c,n,N)$ for $p_r \to 0$ and $n \to \infty$ keeping $m = np_r$ fixed, see HALD (1968b).

In the first part of the paper, Sections 2–8, the problem is discussed for an arbitrary prior distribution. The properties of $d(c,m)$ and $R(c,m,M)$ are derived and it is shown how the optimum plan may be constructed as a function of M. It is proved that the minimum regret, $R(M)$, is increasing and concave, i.e. $R(M)$ is an increasing, continuous and piecewise differentiable function of M with decreasing slope, and that the optimum acceptance number and the optimum sample size are increasing functions of M.

In Section 9 the solution is discussed for a two-point prior distribution. Choosing a convenient normalization the regret function may be written as

$$R^*(c,m,M) = m + (M-m)\{\gamma_1(1-B(c,m)) + \gamma_2 B(c,rm)\},$$

which means that the solution depends on the four parameters M, γ_1, γ_2 and r. It is shown that the optimum plan for (M,γ_1,γ_2,r) is approximately equal to the optimum plan for $(M\gamma_1,1,\gamma_2/\gamma_1,r)$. Therefore, optimum plans are tabulated for $\gamma_1 = 1$ only. The asymptotic solution for $M \to \infty$ is given and it is shown how to use the asymptotic formulas to obtain approximations to the exact solution. Tables of auxiliary constants are given in the Appendix. Asymptotically, m is a linear function of $\ln M - \frac{1}{2}\ln\ln M$ and c is a linear function of m. Finally, it is shown that the Poisson solution may be used as a good approximation to the binomial solution if the break-even quality, p_r, is less than 0.05 and $r = p_2/p_1 > 3$. Even for p_r as large as 0.10 the Poisson approximation will be good for $r > 5$. A similar system of sampling plans has been discussed by WESTERBERG (1964).

Section 10 contains a discussion of the solution for a gamma prior distribution with scale parameter $1/\tau$ and shape parameter s, i.e. expectation $\bar{\lambda} = s/\tau$. The optimum plan depends on four parameters $(M,\lambda_s,s,\bar{\lambda})$, where λ_s represents the costs of sampling inspection divided by the costs of accepting a unit of break-even quality. The case $\lambda_s = 1$ corresponds to rectifying inspection. It is shown that the optimum plan for $(M,\lambda_s,s,\bar{\lambda})$ is approximately equal to the optimum plan for $(M^*,1,s,\bar{\lambda})$ where $M^* = M(1-\lambda_0)/(\lambda_s-\lambda_0)$, λ_0 being an auxiliary constant depending on the prior distribution. Therefore, optimum plans have been tabulated for $\lambda_s = 1$ only. The asymptotic solution for $M \to \infty$ is discussed and used to obtain an approximation to the exact solution. Tables of auxiliary constants are given in the Appendix. Asymptotically, m is a linear function of $\sqrt{M}$ and c is a linear function of m.

The Appendix contains tables of the optimum plans for the two types of prior distributions, the two-point distribution and the gamma distribution.

Throughout the paper we shall reserve the variables x, c, m and M to denote number of defects in the sample, acceptance number, sample size and lot size, respectively. Unless otherwise specified the range of these variables will be $x = 0,1,2, \ldots$, $c = 0,1,2,\ldots$, $0 \leqq m < \infty$, and $0 \leqq M < \infty$.

We shall let Δ denote the usual forward difference operator, $\Delta f(x) = f(x+1) - f(x)$. Differentiation with respect to m will be denoted by a prime when convenient, differentiation with respect to lot size will be indicated by D, D^+ or D^-.

2. The mixed Poisson distribution and the posterior distribution of λ

The mixed Poisson distribution is defined as

$$b_w(x,m) = \int_0^\infty b(x,m\lambda)\,dW(\lambda) \quad \text{for } x = 0,1,\ldots \quad \text{and } m \geqq 0. \tag{4}$$

It represents the marginal probability of getting x defects in a sample of m units, λ being a random variable with distribution function $W(\lambda)$. We shall assume that all moments of the distribution $W(\lambda)$ are finite.

The posterior distribution of λ, i.e. the distribution of λ, given x and m, may be found from

$$dP(\lambda|x,m) = b(x,m\lambda)\,dW(\lambda)/b_w(x,m) \quad \text{for } m > 0. \tag{5}$$

For $m = 0$ the posterior distribution is defined by continuity as

$$dP(\lambda\,|\,x,0) = \lambda^x\,dW(\lambda)/\int_0^\infty \lambda^x\,dW(\lambda).$$

The rth moment becomes

$$E(\lambda^r | x, m) = \int_0^\infty \lambda^{x+r} e^{-m\lambda} dW(\lambda) / \int_0^\infty \lambda^x e^{-m\lambda} dW(\lambda). \tag{6}$$

For $m > 0$ we find

$$E(\lambda^r | x, m) = \frac{(x+r)^{(r)}}{m^r} \frac{b_w(x+r, m)}{b_w(x, m)},$$

where $x^{(r)} = x(x-1) \ldots (x-r+1)$.

Setting $E(\lambda | x,m) = \mu(x,m)$, we get

$$\mu(x, m) = \int_0^\infty \lambda^{x+1} e^{-m\lambda} dW(\lambda) / \int_0^\infty \lambda^x e^{-m\lambda} dW(\lambda), \tag{7}$$

such that

$$E(\lambda^r | x, m) = \mu(x, m)\mu(x+1, m) \ldots \mu(x+r-1, m). \tag{8}$$

We note that a scale transformation of λ in the prior distribution induces a scale transformation of the inspection unit. Put $W_a(\lambda) = W(a\lambda)$ for $a > 0$ and let P_a denote the posterior distribution corresponding to $W_a(\lambda)$. Using (5) we find

$$P_a(\lambda^* | x, am) = \int_0^{\lambda^*} b(x, am\lambda) dW(a\lambda) / \int_0^\infty b(x, am\lambda) dW(a\lambda) = P(a\lambda^* | x, m) \tag{9}$$

and, consequently,

$$a\mu_a(x, am) = \mu(x, m) \tag{10}$$

with $\mu_a(x,m)$ denoting the posterior mean corresponding to the prior distribution W_a.

In the following lemmas we shall study the properties of the posterior mean.

Lemma 1. Let the prior distribution be non-degenerate. The posterior mean, $\mu(x,m)$, is then an increasing function of x and a differentiable, decreasing function of m.

Proof. The assumption implies that the posterior distribution $P(\lambda | x,m)$ is non-degenerate. Hence we have that $V(\lambda | x,m) = E(\lambda^2 | x,m) - (\mu(x,m))^2 > 0$. Since $\mu(x,m) > 0$ we obtain by means of (8) that $\mu(x+1,m) > \mu(x,m)$.

From (7) we find that $\mu(x,m)$ is differentiable with derivative

$$\mu'(x, m) = -E(\lambda^2 | x, m) + (\mu(x, m))^2 = -V(\lambda | x, m) < 0,$$

which shows that $\mu(x,m)$ is a decreasing function of m.

An example has been shown in Figs. 1 and 2.

In the determination of the optimal sampling plans we shall be particularly concerned with samples yielding a posterior mean equal to some specified break-even value. In view of (10) it suffices to discuss the case where the break-even value

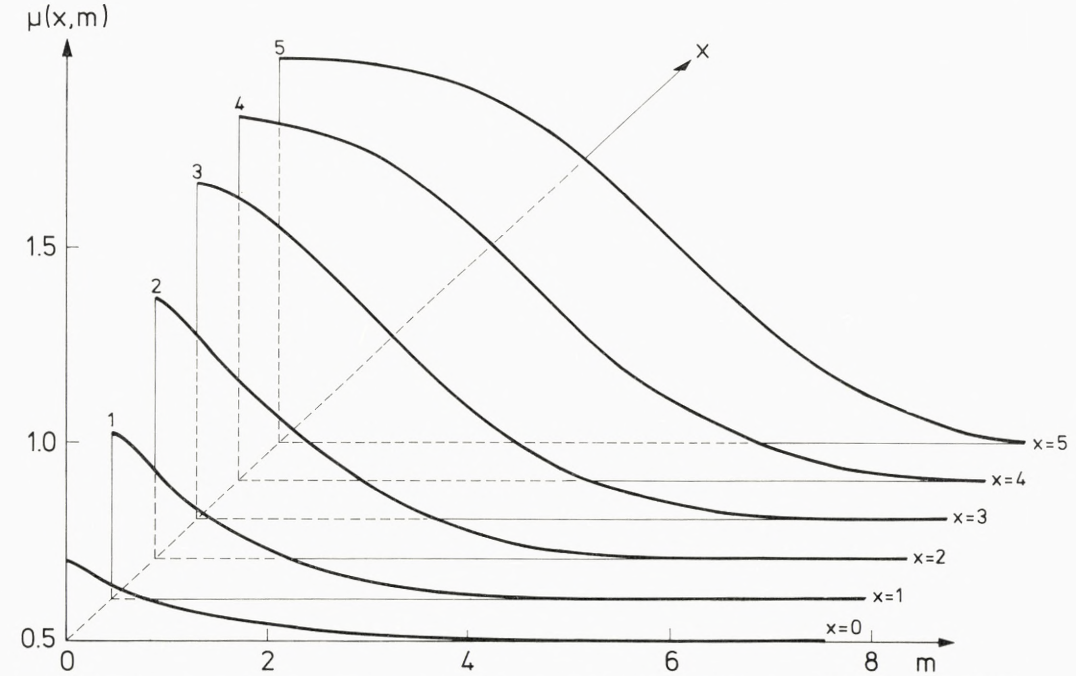

Fig. 1. $\mu(x,m)$ for a two-point prior distribution with $\lambda_1 = 0.5$, $\lambda_2 = 1.5$ and $w_1 = 0.8$.

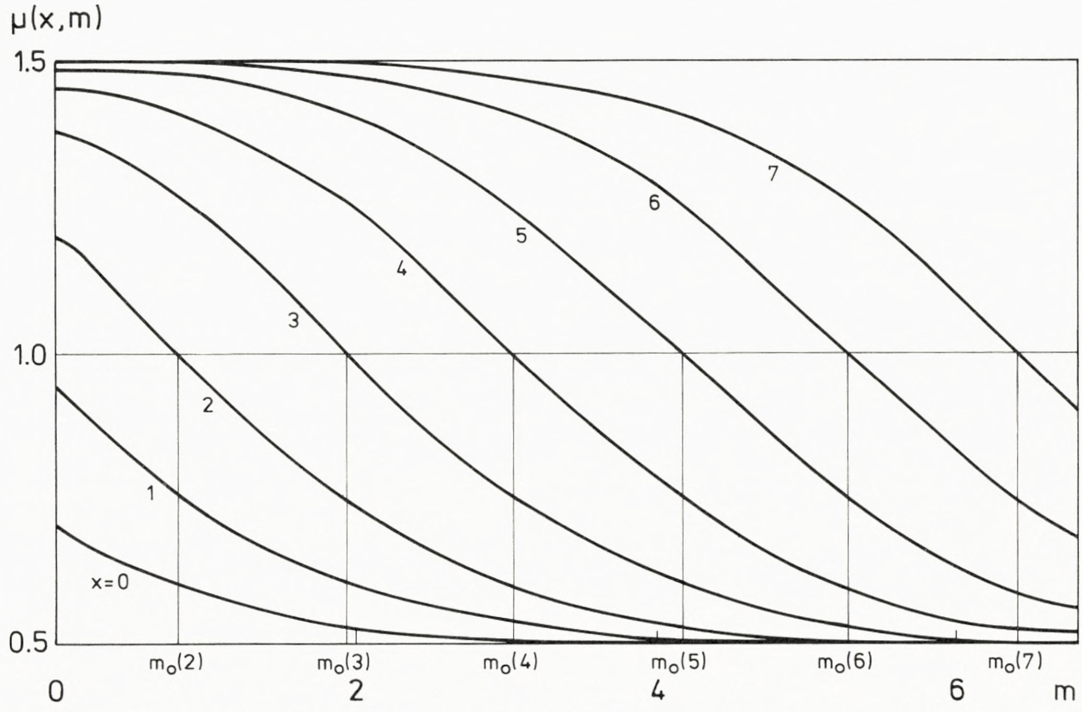

Fig. 2. $\mu(x,m)$ for a two-point prior distribution with $\lambda_1 = 0.5$, $\lambda_2 = 1.5$ and $w_1 = 0.8$.

is equal to 1, say. The following lemma gives the basic properties of the solution to the equation $\mu(x,m) = 1$.

Lemma 2. Let $W(\lambda)$ be such that $W(1) < 1$ and $E\lambda < 1$.
A. Let $c_0(m)$ denote the largest value of x, $x = 0,1,\ldots,$ satisfying $\mu(x,m) \leqq 1$. Then $c_0(m)$ is non-decreasing.
B. Let $m_0(c)$ denote the smallest value of m satisfying $\mu(c,m) \leqq 1$. Then $m_0(c) = 0$ for $c \leqq c_0$ and $m_0(c)$ is increasing for $c > c_0$, where $c_0 = c_0(0)$.

We remark that the definitions lead to the two relations

$$\mu(c_0(m),m) \leqq 1 < \mu(c_0(m)+1,m)$$

and

$$\mu(c,m_0(c)) = 1 \text{ for } c > c_0 .$$

Proof. Let λ^* belong to the spectrum of $W(\lambda)$. If $x \to \infty$ and $m \to \infty$ such that $x/m \to \lambda^*$ then it follows from the law of large numbers for posterior distributions that $\mu(x,m) \to \lambda^*$.

Since $W(1) < 1$ there exists a $\lambda^* > 1$ belonging to the spectrum of W. Hence, for m^* sufficiently large there exists an integer $x^* = [m^*\lambda^*]$, [ ] denoting "the integral part of", such that $\mu(x^*,m^*) > 1$, and since $\mu(x,m)$ is a decreasing function of m we have

$$\mu(x^*,m) > \mu(x^*,m^*) > 1 \text{ for } m < m^*.$$

As

$$\mu(0,m) \leqq \mu(0,0) = E\lambda < 1 \tag{11}$$

we finally get

$$\mu(0,m) < 1 < \mu(x^*,m) \text{ for } m < m^*,$$

which proves the existence of $c_0(m)$ since $\mu(x,m)$ is an increasing function of x. The monotonicity of $c_0(m)$ follows from the monotonicity of $\mu(x,m)$.

To prove the existence of $m_0(c)$ we proceed analogously. For $\lambda^* < 1$ in the spectrum of W we may choose m^* sufficiently large such that $\mu(x^*,m^*) < 1$, where $x^* = [m^*\lambda^*] > c$. Hence, $\mu(c,m^*) < 1$ and thus $m_0(c)$ is well-defined. By definition $m_0(c)$ is non-negative. For $c \leqq c_0$ we have $m_0(c) = 0$ since $\mu(c,0) \leqq 1$. For $c > c_0$ we note that $m_0(c)$ is the solution to the equation $\mu(c,m) = 1$ which shows that $m_0(c)$ is increasing.

An example has been shown in Fig. 3. We note that $c_0(m)$ is a natural extension of the inverse function of $m_0(c)$ since $c_0(m_0(c)) = c$ for $c \geqq c_0$. Moreover the relation $m_0(c) = \min\{m\colon c_0(m) = c\}$ for $c \geqq c_0$ shows that $m_0(c_0(m)) \leqq m$, see Fig. 3.

We shall particularly study two examples.

Let $W(\lambda)$ be a two-point distribution, characterized by the parameters $(\lambda_1, \lambda_2, w_1, w_2)$, $0 < \lambda_1 < 1 < \lambda_2$, and $w_1 + w_2 = 1$. We then have

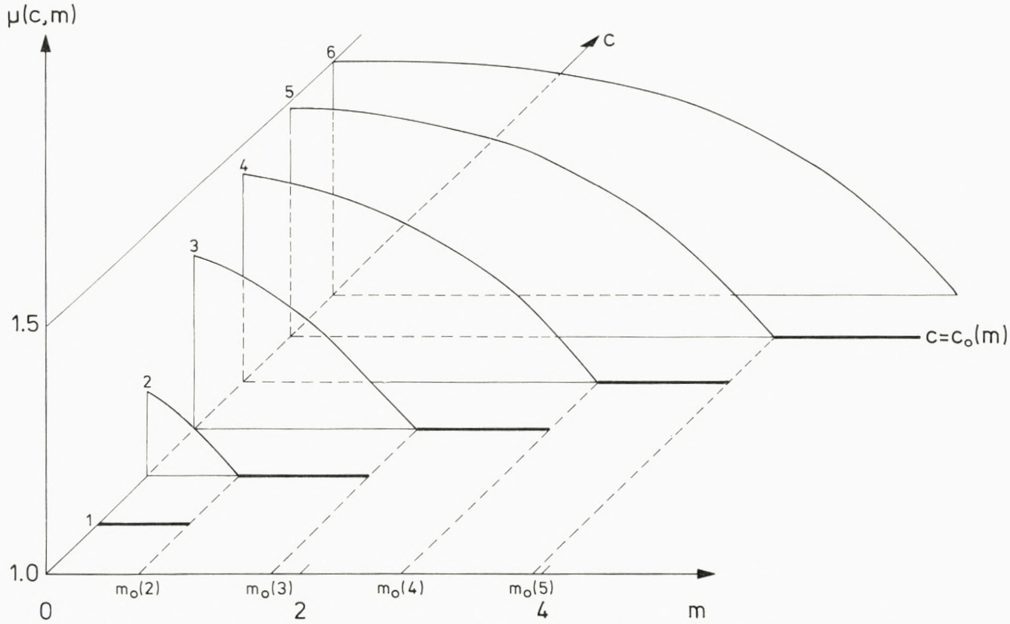

Fig. 3. $c_0(m)$ for a two-point prior distribution with $\lambda_1 = 0.5$, $\lambda_2 = 1.5$ and $w_1 = 0.8$.

$$b_2(x,m) = w_1\ b(x,m\lambda_1) + w_2 b(x,m\lambda_2),$$
$$E(\lambda) = w_1\lambda_1 + w_2\lambda_2$$

and

$$\mu(x,m) = \frac{w_1 e^{-m\lambda_1}\lambda_1^{x+1} + w_2 e^{-m\lambda_2}\lambda_2^{x+1}}{w_1 e^{-m\lambda_1}\lambda_1^{x} + w_2 e^{-m\lambda_2}\lambda_2^{x}}$$
$$= \lambda_1 + (\lambda_2 - \lambda_1)\left\{1 + \frac{w_1}{w_2}\left(\frac{\lambda_1}{\lambda_2}\right)^x e^{m(\lambda_2-\lambda_1)}\right\}^{-1}.$$

We remark that $\lambda_1 \leqq \mu(x,m) \leqq \lambda_2$.

The condition $\mu(x,m) \leqq 1$ is equivalent to

$$\ln\frac{w_2(\lambda_2 - 1)}{w_1(1-\lambda_1)} \leqq x\ln\frac{\lambda_1}{\lambda_2} + m(\lambda_2 - \lambda_1),$$

which leads to

$$c_0(m) = \left\{\begin{array}{ll} 0 & \text{for } m < (\lambda_2-\lambda_1)^{-1}\ln\dfrac{w_2(\lambda_2-1)}{w_1(1-\lambda_1)} \\ \left[\left\{m(\lambda_2-\lambda_1) - \ln\dfrac{w_2(\lambda_2-1)}{w_1(1-\lambda_1)}\right\}/\ln\dfrac{\lambda_2}{\lambda_1}\right] & \text{otherwise} \end{array}\right\} \quad (12)$$

and

$$m_0(c) = \begin{cases} \dfrac{1}{\lambda_2 - \lambda_1}\left\{\ln\dfrac{w_2(\lambda_2-1)}{w_1(1-\lambda_1)} + c\ln\dfrac{\lambda_2}{\lambda_1}\right\} & \text{for } c > c_0 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

As another example we shall let $W(\lambda)$ be a gamma distribution with parameters (s,τ) where $s < \tau$, i.e.

$$dW(\lambda) = e^{-\tau\lambda}(\tau\lambda)^{s-1}d(\tau\lambda)/\Gamma(s), \tag{14}$$

so that $E(\lambda) = s/\tau < 1$,

$$b_\gamma(x,m) = \frac{\Gamma(s+x)}{\Gamma(s)\Gamma(x+1)}\,\frac{\tau^s m^x}{(\tau+m)^{s+x}} \tag{15}$$

and

$$\mu(x,m) = \frac{s+x}{\tau+m}. \tag{16}$$

It is seen from (16) that the parameters s and τ may be interpreted as the number of defects, s, found in τ units (observed a priori) and that these numbers combine additively with x and m in determining the posterior mean.

From (16) we find

$$c_0(m) = [m + \tau - s] \tag{17}$$

and

$$m_0(c) = \begin{cases} c + s - \tau & \text{for } c > [\tau - s] \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

For a general discussion of the mixed Poisson distribution the reader is referred to the monograph by LUNDBERG (1940, 1964).

3. The model

Let the cost of sampling inspection, acceptance and rejection per unit of product be $k_s(\lambda) = S_1 + S_2\lambda$, $k_a(\lambda) = A_1 + A_2\lambda$ and $k_r(\lambda) = R_1 + R_2\lambda$, respectively. The coefficients are assumed to be non-negative, and the break-even quality, defined as solution to the equation $k_a(\lambda) = k_r(\lambda)$, is supposed to be equal to 1, if the contrary is not explicity stated, i.e. $R_1 - A_1 = A_2 - R_2$, where $R_1 > A_1 \geqq 0$ and $A_2 > R_2 \geqq 0$. This may always be achieved by a suitable choice of observational unit. Hence, a process with mean occurrence rate of defects less than or equal to 1 should be accepted whereas for $\lambda > 1$ the process should be rejected.

We shall also assume that $S_1 \geqq R_1$ and $S_2 \geqq R_2$ so that $k_s(\lambda) \geqq k_r(\lambda)$.

Taking the prior distribution $W(\lambda)$ into account we find that the expected cost of accepting a lot of M units is $Mk_a(\bar{\lambda})$ and the expected cost of rejection is $Mk_r(\bar{\lambda})$, where $\bar{\lambda} = E(\lambda)$. We shall assume that $\bar{\lambda} < 1$ so that on the average it is cheaper to accept than to reject a lot. We shall further exclude the trivial case $W(1) = 1$, since in this case all lots should be accepted.

The minimum (unavoidable) costs per unit are composed of acceptance costs for $\lambda \leqq 1$ and rejection costs for $\lambda > 1$, i.e.

$$k_0 = \int_0^1 k_a(\lambda)\,dW(\lambda) + \int_1^\infty k_r(\lambda)\,dW(\lambda). \tag{19}$$

By sampling inspection we inspect a random sample of m units out of a lot of M units, where $0 < m < M$. If the number of defects observed is less than or equal to c, the remainder of the lot is accepted, otherwise it is rejected. The expected cost for this procedure becomes

$$K = mk_s(\bar{\lambda}) + (M-m)\int_0^\infty \{k_a(\lambda)P(\lambda) + k_r(\lambda)Q(\lambda)\}\,dW(\lambda),$$

where $P(\lambda) = B(c,m\lambda)$ and $Q(\lambda) = 1 - P(\lambda)$. The optimum sampling plan is determined by minimizing K with respect to (c,m). It will be shown that the minimum exists. If $\min K < Mk_a(\bar{\lambda})$ sampling inspection is preferred, otherwise acceptance without inspection should be used.

In the following discussion of the decision procedure we shall use decision (or opportunity) losses rather than costs. We therefore introduce the difference

$$\left.\begin{aligned} K - Mk_0 &= m\{k_s(\bar{\lambda}) - k_0\} \\ &+ (M-m)(A_2 - R_2)\left\{\int_0^1 (1-\lambda)Q(\lambda)dW(\lambda) + \int_1^\infty (\lambda - 1)P(\lambda)dW(\lambda)\right\}. \end{aligned}\right\} \tag{20}$$

Instead of minimizing K we may just as well minimize the expected (standardized) regret to find the optimum procedure. The expected regret is defined as

$$R = (K - Mk_0)/(A_2 - R_2). \tag{21}$$

By proper standardization we have thus transformed the original problem to one with the special cost functions $k_a(\lambda) = \lambda$ and $k_r(\lambda) = 1$. To simplify R we introduce

$$\lambda_0 = \int_0^1 \lambda dW(\lambda) + \int_1^\infty dW(\lambda), \tag{22}$$

the minimum costs for the special cost functions, and

$$\delta = \lambda_s - \lambda_0 = \{k_s(\bar{\lambda}) - k_0\}/(A_2 - R_2), \tag{23}$$

which leads to

$$\left.\begin{aligned}\lambda_s &= \{S_1 - A_1 + (S_2 - R_2)\bar{\lambda}\}/(A_2 - R_2)\\ &= 1 + \{k_s(\bar{\lambda}) - k_r(\bar{\lambda})\}/(A_2 - R_2) \geqq 1.\end{aligned}\right\} \tag{24}$$

We note that

$$\delta_0 = 1 - \lambda_0 = \int_0^1 (1-\lambda)\,dW(\lambda) > 0 \tag{25}$$

and

$$\bar{\lambda} - \lambda_0 = \int_1^\infty (\lambda - 1)\,dW(\lambda) > 0$$

represent the (standardized) loss per item for the two singular decisions, viz. rejection without inspection and acceptance without inspection.

From (20), (21) and (23) we find

$$R(c,m,M) = m\delta + (M-m)\ d(c,m) \qquad \text{for } 0 < m < M, \tag{26}$$

where

$$d(c,m) = \int_0^1 (1-\lambda)Q(\lambda)\,dW(\lambda) + \int_1^\infty (\lambda - 1)P(\lambda)\,dW(\lambda). \tag{27}$$

Since $|\lambda - 1|$ represents the standardized (opportunity) loss, and $P(\lambda)$ denotes the acceptance probability (the operating characteristic), $d(c,m)$ gives the expected decision loss per unit.

Using (25) we find

$$d(c,m) = \delta_0 - \int_0^\infty (1-\lambda)P(\lambda)\,dW(\lambda), \tag{28}$$

where δ_0 is independent of the sampling plan. Introducing the posterior distribution of λ in the last term of (28) we get

$$\int_0^\infty (1-\lambda)P(\lambda)\,dW(\lambda) = \sum_{x=0}^{c} \{1 - \mu(x,m)\}\,b_w(x,m). \tag{29}$$

Putting $m = 0$ in (28) we find

$$d(c,0) = \delta_0 - \int_0^\infty (1-\lambda)\,dW(\lambda) = \bar{\lambda} - \lambda_0, \tag{30}$$

such that the regret for acceptance without inspection is

$$R_a(M) = M\int_1^\infty (\lambda - 1)\,dW(\lambda) = Md(c,0) = R(c,0,M).$$

Letting $m \to \infty$ in (28) we get

$$d(c,\infty) = \delta_0, \tag{31}$$

which shows that the regret for rejection without inspection is

$$R_r(M) = M\int_0^1 (1-\lambda)\,dW(\lambda) = Md(c,\infty).$$

Sampling inspection is preferred if $\min\limits_{c\geq 0,\, m>0} R\ (c,m,M) < R_a(M)$.

The model has the important property that the expected regret (26) for a fixed sampling plan (c,m), say, is a linear function of M for $m < M$. To simplify the subsequent discussion of the optimal sampling plan we shall formally extend the domain of definition for R by setting

$$R(c,m,M) = M\delta \quad \text{for } 0 \leqq M \leqq m. \tag{32}$$

Using this convention the regret becomes a continuous function of M for $0 \leqq M < \infty$, consisting of two linear segments. We shall now prove that $R(c,m,M)$ is concave, i.e. we shall prove that $d(c,m) \leqq \delta$.

From (28) we find that the derivative of $d(c,m)$ is

$$\left.\begin{aligned} d'(c,m) &= \int_0^\infty (1-\lambda)\lambda b(c,m\lambda)\,dW(\lambda) \\ &= b_w(c,m)E(\lambda-\lambda^2\,|\,c,m) \\ &= b_w(c,m)\mu(c,m)\{1-\mu(c+1,m)\}. \end{aligned}\right\} \tag{33}$$

Combing (33) with the result stated in Lemma 1, it is easily seen that $d(c,m)$ is decreasing for $0 \leqq m \leqq m_0(c+1)$ and that $d(c,m)$ is increasing for $m_0(c+1) \leqq m$. If $m_0(c+1) = 0$, the statement simply means that $d(c,m)$ is increasing. Consequently we find

$$\left.\begin{aligned} d(c,m) &\leqq \max\{d(c,0), d(c,\infty)\} \\ &= \max\{\bar{\lambda}-\lambda_0, 1-\lambda_0\} = \delta_0 \leqq \delta, \end{aligned}\right\} \tag{34}$$

where the last inequality follows from (23) and (24). In Section 4 we shall give a more detailed discussion of $d(c,m)$.

For any sampling plan the regret is thus a concave function of M consisting of two linear segments intersecting at the point $(m,\delta m)$. For $m = 0$ the regret is the linear function $R_a(M) = M(\bar{\lambda}-\lambda_0)$, corresponding to acceptance without inspection.

It should be noted that with the extended definition given above $R(c,m,M)$ is defined for $c = 0,1,\ldots,$ $0 \leqq m < \infty$ and $0 \leqq M < \infty$.

As mentioned in Section 1 the model may be considered as a limiting case of the binomial model which has been discussed previously, see Hald (1960, 1967a, 1968b).

4. The expected decision loss per unit

Theorem 1.

A. For $c < c_0$ and $m > 0$ we have $d'(c,m) > 0$ and $\Delta d(c,m) < 0$.

B. For $c \geqq c_0$ and $m > 0$ we have $d'(c,m) \lesseqgtr 0$ for $m \lesseqgtr m_0(c+1)$. Further $\Delta d(c,m) > 0$ for $c \geqq c_0(m)$ and $\Delta d(c,m) \leqq 0$ for $c < c_0(m)$ with strict inequality holding except for $m = m_0(c+1)$.

C. $d(c,0) = \bar{\lambda} - \lambda_0$, $d'(0,0) = \bar{\lambda} - E\lambda^2$ and $d'(c,0) = 0$ for $c > 0$.

Proof. We shall first prove the properties of $d'(c,m)$. From (33) we have $d'(c,m) = b_w(c,m)\, \mu(c,m)\{1 - \mu(c+1,m)\}$ so that for $m > 0$

$$d'(c,m) \lesseqgtr 0 \text{ for } \mu(c+1,m) \gtreqless 1.$$

For $c < c_0$ we have $\mu(c+1,m) \leqq \mu(c_0,m) < \mu(c_0,0)$ according to Lemma 1. Furthermore, the definition of c_0 implies that $\mu(c_0,0) \leqq 1$, and hence we have $\mu(c+1,m) < 1$, which shows that $d'(c,m) > 0$. For $c \geqq c_0$ and $m > 0$ we find $\mu(c+1,m) \gtreqless 1$ for $m \lesseqgtr m_0(c+1)$ and consequently $d'(c,m) \lesseqgtr 0$ for $m \lesseqgtr m_0(c+1)$.

For $m = 0$ we note that $b_w(c,m) = 0$ except in the case $c = 0$ where $b_w(0,0) = 1$. Using (7) we find $\mu(0,0)\{1 - \mu(1,0)\} = \bar{\lambda} - E\lambda^2$ which proves the last statement regarding $d'(c,m)$.

Since

$$\left.\begin{aligned} \Delta d(c,m) &= \int_0^\infty (\lambda - 1)\, b(c+1, m\lambda)\, dW(\lambda) \\ &= b_w(c+1,m)\{\mu(c+1,m) - 1\}, \end{aligned}\right\} \quad (35)$$

the statements regarding $\Delta d(c,m)$ are proved analogously. We remark that (35) and (33) lead to $\Delta d(c,m) = -d'(c,m)\, m/(c+1)$, and hence $\Delta d(c,m)$ and $d'(c,m)$ have opposite signs. This concludes the proof.

As immediate consequences of the theorem we find that $\min_c d(c,m) = d(c_0(m),m)$ and $\min_m d(c,m) = d(c, m_0(c+1))$. These results are intuitively reasonable since $c_0(m)$ and $m_0(c)$ are the values of c and m, respectively, yielding a posterior mean close to the break-even quality.

We shall prove that $\min_c d(c,m) = d(c_0(m),m)$ is a decreasing and continuous function of m which is differentiable except for $m = m_0(c)$, $c = c_0 + 1, c_0 + 2, \ldots$, see Fig. 4. Let us first consider the open intervals $m_0(c) < m < m_0(c+1)$ for $c \geqq c_0$. We then have $c_0(m) = c$, and accordingly we find that $\min_x d(x,m) = d(c,m)$ which is differentiable. Hence $\frac{\partial}{\partial m} \min_x d(x,m) = d'(c,m)$ which is negative since $m < m_0(c+1)$. Next we consider the endpoints $m = m_0(c)$. For $c > c_0$ and $m \uparrow m_0(c)$ we have $\min_x d(x,m) \to d(c-1, m_0(c))$ and $\frac{\partial}{\partial m} \min_x d(x,m) \to d'(c-1, m_0(c)) = 0$. For $m \downarrow m_0(c)$

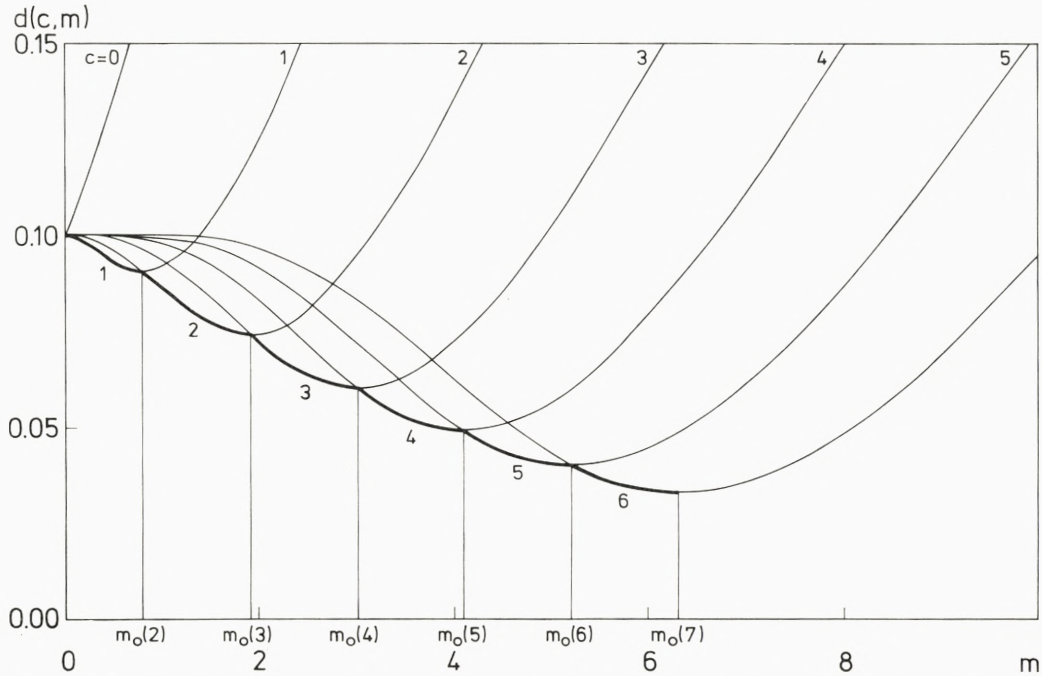

Fig. 4. $d(c,m)$ for a two-point prior distribution with $\lambda_1 = 0.5$, $\lambda_2 = 1.5$ and $w_1 = 0.8$.

with $c \geqq c_0$ we get $\min\limits_x d(x,m) \to d(c,m_0(c))$ and $\dfrac{\partial}{\partial m} \min\limits_x d(x,m) \to d'(c,m_0(c))$. Since $d(c-1,m_0(c)) = d(c,m_0(c))$ the statement is proved.

To study $d(c,m)$ further we also investigate $d''(c,m)$. For $m > 0$ we find $b'(c,m\lambda) = cm^{-1}\,b(c,m\lambda) - (c+1)\,m^{-1}\,b(c+1,m\lambda)$, such that

$$\left.\begin{aligned} d''(c,m) &= cm^{-1}\int_0^\infty (1-\lambda)\lambda b(c,m\lambda)\,dW(\lambda) \\ &- (c+1)m^{-1}\int_0^\infty (1-\lambda)\lambda b(c+1,m\lambda)dW(\lambda) \\ &= cm^{-1}d'(c,m) - (c+1)m^{-1}d'(c+1,m). \end{aligned}\right\} \quad (36)$$

For $m = 0$ we find

$$d''(c,0) = \left\{\begin{array}{ll} E\lambda^3 - E\lambda^2 & \text{for } c = 0 \\ E\lambda^2 - E\lambda^3 & \text{for } c = 1 \\ 0 & \text{for } c > 1. \end{array}\right\} \quad (37)$$

For $m = m_0(c+1)$ and $c \geqq c_0$, we have by Theorem 1

$$d''(c,m) = -(c+1)m^{-1}d'(c+1,m) > 0,$$

which shows that $d(c,m)$ is convex in the neighbourhood of $m_0(c+1)$.

Consider the interval where $d'(c,m) < 0$, i.e. $0 < m < m_0(c+1)$. In Section 6 we use the assumption that $d''(c,m)$ changes sign at most once in this interval and that $\log(-d'(c,m))$ is concave in the subinterval where $d(c,m)$ is concave, i.e. $\{\log(-d')\}'' < 0$ if $d'' < 0$. We have not succeeded in finding general conditions for this to be true. For the two examples previously discussed, however, $d(c,m)$ has the property required, as will be shown below. We shall use the notation

$$l(c,m) = \frac{\partial}{\partial m}\log(-d'(c,m)) = d''(c,m)/d'(c,m).$$

For the two-point prior distribution we find for $0 < m < m_0(c+1)$ and $c \geqq c_0$

$$-d'(c,m) = \frac{m^c}{c!}w_1\lambda_1^{c+1}(1-\lambda_1)\exp(-m\lambda_1)[\exp\{(\lambda_2-\lambda_1)(m_0(c+1)-m)\}-1], \quad (38)$$

and consequently

$$l'(c,m) = -\frac{c}{m^2} - \frac{(\lambda_2-\lambda_1)^2\exp\{(\lambda_2-\lambda_1)(m_0(c+1)-m)\}}{[\exp\{(\lambda_2-\lambda_1)(m_0(c+1)-m)\}-1]^2}$$

which is negative. We have thus proved that $\log(-d'(c,m))$ is concave in $(0, m_0(c+1))$. Since $l(c,m) = d''/d'$ it is easily verified that $d''(c,m)$ changes sign at most once for $0 < m < m_0(c+1)$.

For the gamma prior we find from (33), (15) and (16)

$$-d'(c,m) = \frac{\Gamma(s+c+1)}{\Gamma(s)\Gamma(c+1)}\frac{\tau^s m^c}{(\tau+m)^{s+c+1}}\left\{\frac{s+c+1}{\tau+m}-1\right\} \quad (39)$$

and accordingly

$$l(c,m) = -\frac{m_0(c+1)+\tau+1}{m+\tau} - \frac{1}{m_0(c+1)-m} + \frac{c}{m}, \quad (40)$$

which may be written as

$$l(c,m) = \frac{(s+1)(m-m')(m-m'')}{m(m+\tau)(m_0(c+1)-m)}, \quad (41)$$

where m' and m'' denote the roots of the second degree polynomial in the numerator. It is easy to verify that both roots are positive.

From (40) it follows that $l(c,0+) = +\infty$ and $l(c,m_0(c+1)) = -\infty$. Hence, $l(c,m)$ equals zero an odd number of times in $(0, m_0(c+1))$. However, since (41) shows that $l(c,m)$ equals zero exactly two times in $(0,\infty)$, we must have $m' < m_0(c+1) < m''$, which shows that $d''(c,m)$ changes sign exactly once in $(0, m_0(c+1))$.

Consider now $l(c,m)$ for $0 < m < m'$. Logarithmic differentiation of (41) gives

$$l'/l = -(m'-m)^{-1} - (m''-m)^{-1} - m^{-1} - (m+\tau)^{-1} + (m_0(c+1)-m)^{-1}.$$

Since $m'-m < m_0(c+1)-m$ we have that $l'/l < 0$, and as $l(c,m) > 0$ we get $l'(c,m) < 0$, showing that $\log(-d'(c,m))$ is concave for $0 < m < m'$, which is the interval where $d(c,m)$ is concave.

5. The regret as function of the acceptance number

Suppose that the sample size has been decided upon so that the problem is to determine the value of c minimizing $R(c,m,M)$ for given m and M with $0 < m < M$. (If $m = 0$ or $m \geqq M$ the regret is independent of c, and the optimization problem admits any solution).

From (26) we find for $0 < m < M$ that

$$\Delta R(c,m,M) = (M-m)\Delta d(c,m), \tag{42}$$

reflecting the fact that the regret depends on the acceptance number only through the decision loss, $d(c,m)$ which is independent of M. The optimal acceptance number will therefore be independent of lot size. Moreover, for $c_0(m) > 0$ we find from Theorem 1 that $d(c,m)$ is decreasing as function of c for $c < c_0(m)$ and increasing for $c > c_0(m)$. For $c_0(m) = 0$ we find analogously that $d(c,m)$ is increasing.

Thus, using (42) and Theorem 1 we get

Theorem 2.

A. For fixed (m,M) and $0 < m < M$ we have

$$\min_c R(c,m,M) = R(c_0(m),m,M), \tag{43}$$

where $c_0(m)$ is defined in Lemma 2 and $c_0(m) \geqq c_0$. $R(c,m,M)$ is an increasing function of c if $c_0(m) = 0$, whereas R is first decreasing and then increasing if $c_0(m) > 0$. For $m = m_0(c_0(m)+1)$, i.e. $\mu(c_0(m)+1,m) = 1$, we have $R(c_0(m),m,M) = R(c_0(m)+1,m,M)$, otherwise the representation (43) is unique.

B. For $m = 0$ and $M \leqq m$ the regret is independent of c.

The optimal acceptance number is thus given by $c_0(m)$, which is a non-decreasing function of m, see Lemma 2. Accordingly, the set of possible sample sizes, $0 < m < M$, is divided into consecutive intervals by the condition $c_0(m) = c$, $c = c_0, c_0+1, \ldots$, the division points being $m = m_0(c)$, $c = c_0, c_0+1, \ldots$. For $m_0(c) \leqq m < m_0(c+1)$ the optimal acceptance number is c.

6. The regret as function of the sample size

In the discussion of $\inf_m R(c,m,M)$ in the present section we shall make use of the fact that R is a concave function of M for $0 \leqq M < \infty$. Most of the statements regarding the properties of the minimum regret are based on well-known results from the theory of concave functions and are therefore given without proofs. The reader is referred to any standard textbook on convex analysis. Our terminology will follow the monograph of ROCKAFELLER (1970). We shall define

$$R(c,M) = \inf_m R(c,m,M). \tag{44}$$

Theorem 3. Let c be fixed and let $R(c,M)$ be defined by (44). Then we have

A. $R(c,M)$ is an increasing and concave function of M.

B. There exists at least one value of m, $m = m(c,M)$ say, satisfying $R(c,m,M) = R(c,M)$. For any version of $m(c,M)$ we have $m(c,M) = 0$ for $c < c_0$ and $0 \leqq m(c,M) < m_0(c+1)$ for $c \geqq c_0$. If $m(c,M) > 0$ then $R(c,m,M)$ has a local minimum for $m = m(c,M)$.

C. $D^-R(c,M)$ and $D^+R(c,M)$ both exist and are non-increasing functions of M. Furthermore we have

$$d(c,m(c,M-)) = D^-R(c,M) \geqq d(c,m(c,M)) \geqq D^+R(c,M) = d(c,m(c,M+)) \tag{45}$$

with equality signs holding except for at most countably many values of M.

D. $m(c,M)$ is a non-decreasing function of M.

Proof. A. Since, for any m, the function $R(c,m,M)$ is concave it follows that the pointwise infimum is again a concave function, defined for $0 \leqq M < \infty$. Therefore, $R(c,M)$ is continuos. That $R(c,M)$ is increasing follows from (45) since $d(c,m) > 0$.

B. Consider now $R(c,m,M)$ as function of m. We note that R is differentiable for $0 < m < M$ and $R(c,m,M) = R(c,M,M)$ for $m \geqq M$. Hence, the infimum is attained for at least one value of m in the interval $0 \leqq m \leqq M$. Moreover, from (30) and (32) we find

$$R(c,0,M) = M(\bar{\lambda} - \lambda_0) < M\delta = R(c,M,M),$$

and hence $\min R(c,m,M)$ is attained either at the leftmost boundary, $m = 0$, corresponding to acceptance without inspection, or at a point $0 < m < M$ yielding a local minimum for R. Differentiating (26) with respect to m we find

$$R'(c,m,M) = \delta - d(c,m) + (M-m)d'(c,m) \tag{46}$$

with $d'(c,m)$ given in (33). For $c < c_0$ we have $d'(c,m) > 0$ by Theorem 1. Moreover (34) shows that $\delta - d(c,m) \geqq 0$ and hence R is increasing. Accordingly, in the case $c < c_0$ we find that the minimum is attained for $m = 0$.

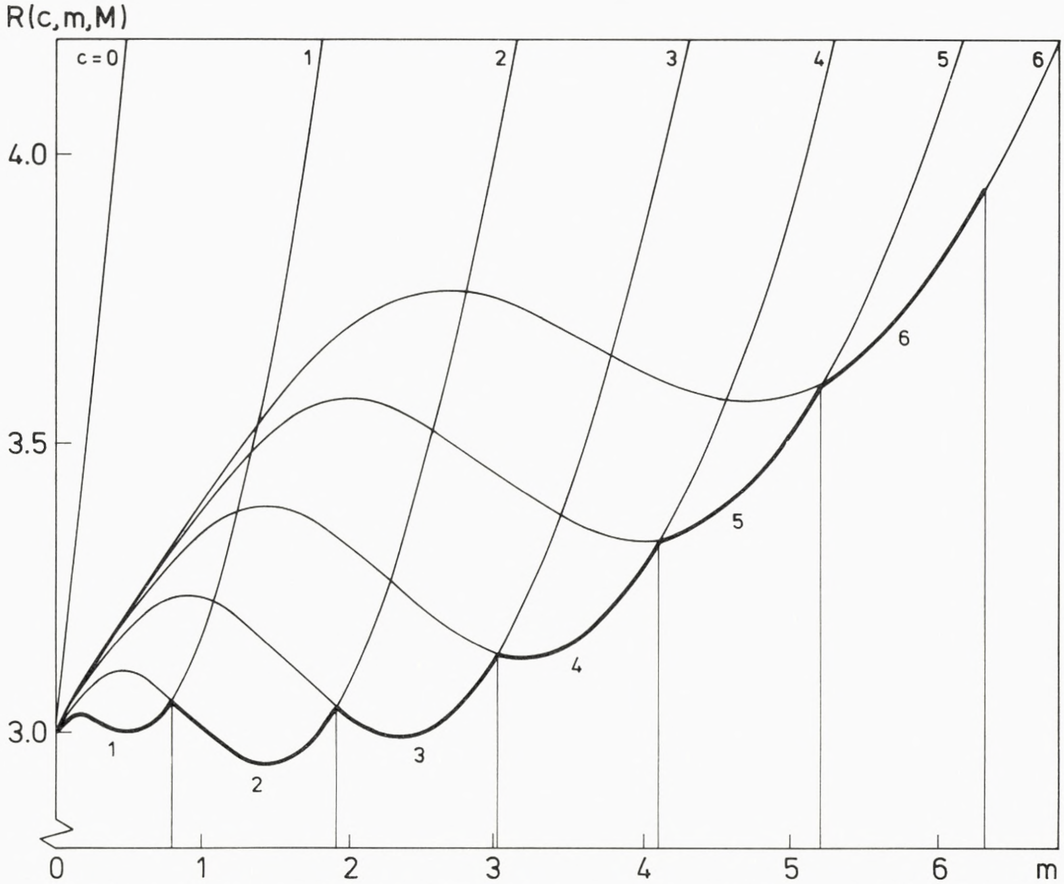

Fig. 5. $R(c,m,M)$ for a two-point prior distribution with $\lambda_1 = 0.5$, $\lambda_2 = 1.5$ and $w_1 = 0.8$. $M = 30$ and $\delta = 0.5$.

Assume now that $c \geqq c_0$ and $M > m_0(c+1)$. (If $M \leqq m_0(c+1)$ it is clear that $0 \leqq m(c,M) < m_0(c+1)$ since $\min R$ is attained for $0 < m < M$). From Theorem 1 we find $d'(c,m) > 0$ for $m > m_0(c+1)$ which shows that R is non-decreasing for $m > m_0(c+1)$, see Figure 5. Thus we have $0 \leqq m(c,M) < m_0(c+1)$.

C. We first note that since $R(c,M)$ is concave we have

$$D^-R(c,M) \geqq D^+R(c,M), \tag{47}$$

where both derivatives exist and are finite, non-increasing functions of M. Moreover, the equality sign holds except for countably many values of M.

Let now M_1 be arbitrary, but fixed, and let $m_1 = m(c,M_1)$ for some version of $m(c,M)$. Since $R(c,M)$ is the minimal regret we have

$$\delta m_1 + (M - m_1)\, d(c,m_1) \geqq R(c,M) \tag{48}$$

with equality sign holding for $M = M_1$. Hence, the line determined by the left-hand side of (48) is a line of support for the function $R(c,M)$. Accordingly we have that $d(c,m(c,M))$ is a subgradient of $R(c,M)$ which implies (45).

D. Finally, since $d(c,m)$ is decreasing and since the composite function $d(c,m(c,M))$ is non-increasing we find that $m(c,M)$ is non-decreasing which completes the proof of the theorem.

We remark that $m(c,M)$ is uniquely determined except for at most countably many values of M. Moreover, in most cases $m(c,M)$ will have only one point of ambiguity, viz. for the value of M where the regret for acceptance without inspection equals the local minimum of $R(c,m,M)$, see Fig. 6.

For the two cases discussed previously the following corollary applies.

Corollary 1. Let $c \geqq c_0$ and $0 < m < m_0(c+1)$. If $d''(c,m)$ changes sign at most once and if $\{\log(-d'(c,m))\}'' < 0$ in the subinterval where $d''(c,m) < 0$ then $R(c,M)$ is differentiable except for at most one value of M satisfying

$$R(c,M) = R(c,0,M) = R(c,m,M) \tag{49}$$

for some m, $0 < m < m_0(c+1)$.

Proof. It suffices to prove that $m(c,M)$ is unique and continuous except for at most one value of M. The differentiability will then follow from (45). Since, by the theorem, any version of $m(c,M)$ is non-decreasing it follows that we cannot have $m(c,M_1) > 0$ and $m(c,M_2) = 0$ for $M_1 < M_2$. Thus, if for some M_1 the minimal regret corresponds to a local minimum then this will be true for any $M > M_1$. Hence, it suffices to prove that the value of m leading to a local minimum, if it exists, is a unique continuous function of M. We shall therefore discuss the determination of the local minimum of R.

$R(c,m,M)$ has a local minimum for $m = m_1$ if and only if $R'(c,m_1,M) = 0$ and R' is increasing in the neighbourhood of m_1. For $0 < m < m_0(c+1)$ we have $d'(c,m) < 0$ and we may therefore write (46) as

$$R'(c,m,M) = -d'(c,m)(F(c,m) - M), \tag{50}$$

where

$$F(c,m) = m - \{\delta - d(c,m)\}/d'(c,m) \quad \text{for } 0 < m < m_0(c+1). \tag{51}$$

A typical graph of $F(c,m)$ has been given in Fig. 7. We note that since $d'(c,m) \uparrow 0$ for $m \uparrow m_0(c+1)$ we have $F(c,m) \to \infty$ for $m \uparrow m_0(c+1)$.

Differentiating (51) with respect to m we find

$$F'(c,m) = 2 + d''(c,m)\{\delta - d(c,m)\}/(d'(c,m))^2. \tag{52}$$

We shall show that under the assumptions of the corollary F' will have at most one change of sign.

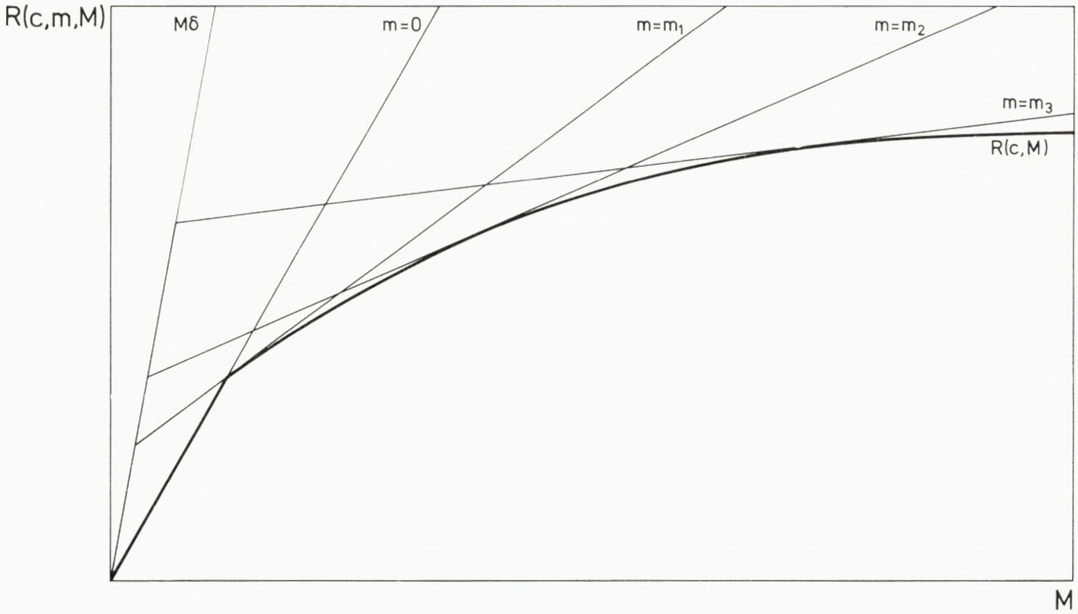

Fig. 6. $R(c,m,M)$ for four values of m, $0 < m_1 < m_2 < m_3$. $R(c,M) = \inf R(c,m,M)$.

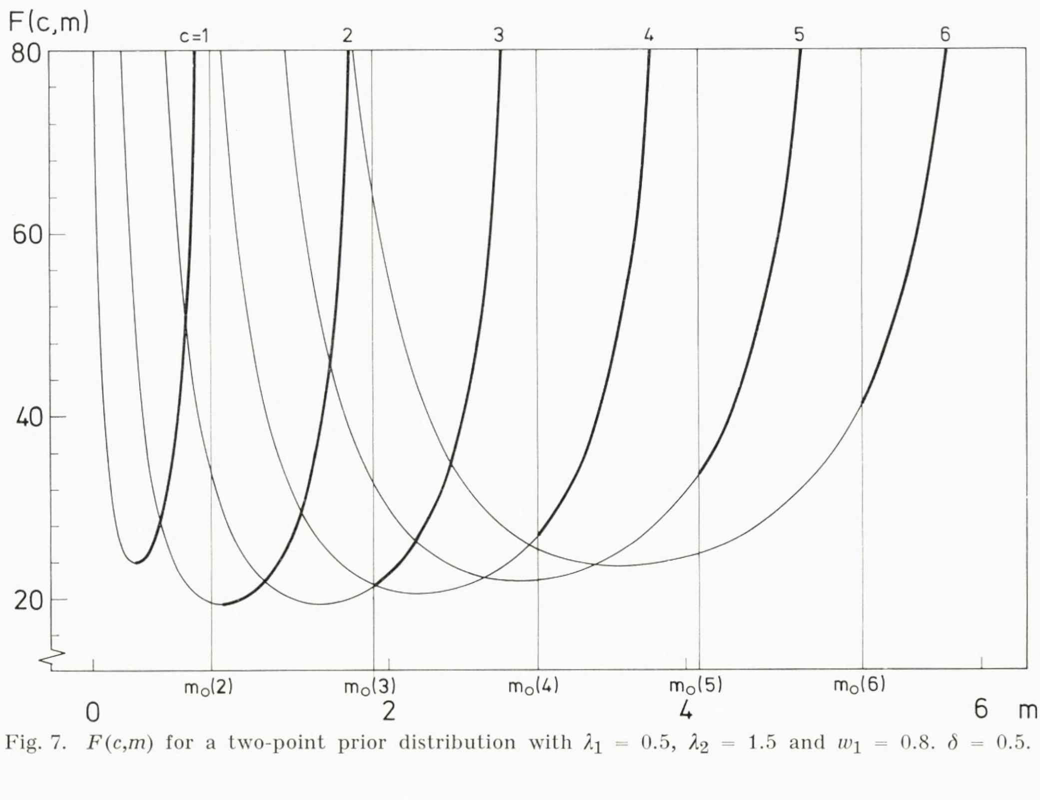

Fig. 7. $F(c,m)$ for a two-point prior distribution with $\lambda_1 = 0.5$, $\lambda_2 = 1.5$ and $w_1 = 0.8$. $\delta = 0.5$.

From (52) we see that $F'(c,m) > 0$ if $d''(c,m) \geqq 0$. For $d''(c,m) < 0$ we have

$$F'(c,m) = d''(c,m)\, l_1(c,m)/(d'(c,m))^2, \tag{53}$$

where

$$l_1(c,m) = 2(d'(c,m))^2/d''(c,m) + \delta - d(c,m)$$

and

$$l'_1(c,m) = d'(c,m)\{3(d''(c,m))^2 - 2d'(c,m)d'''(c,m)\}/(d''(c,m))^2. \tag{54}$$

By assumption we have that $\{\log(-d'(c,m))\}'' < 0$ so that

$$(d''(c,m))^2 - d'(c,m)\, d'''(c,m) > 0$$

which leads to

$$3(d''(c,m))^2 - 2d'(c,m)\, d'''(c,m) > 0.$$

Since $d'(c,m) < 0$ it follows from (54) that l_1 is decreasing and therefore l_1 and F' will equal zero at most once in the interval where $d'' < 0$.

Combining the results for $d'' < 0$ and $d'' \geqq 0$ we find that F has at most two intervals of monotonicity which means that the equation

$$M = F(c,m) \tag{55}$$

will have at most two solutions in the interval $0 < m < m_0(c+1)$. Consequently we find from (50) that R' equals zero at most two times, corresponding to the two solutions of (55).

Differentiating (50) we find

$$R''(c,m,M) = -d''(c,m)(F(c,m) - M) - d'(c,m)F'(c,m). \tag{56}$$

Let now m_1 satisfy $R'(c,m_1,M) = 0$, i.e. $M = F(c,m_1)$. From (56) it then follows that $R''(c,m_1,M) > 0$ if and only if $F'(c,m_1) > 0$ which shows that only (m,M) combinations corresponding to the increasing arc of $F(c,m)$ will yield a local minimum of R. Hence, the local minimum is unique and moreover, the sample size leading to the local minimum is a continuous function of the lot size determined by the increasing arc of the graph of $F(c,m)$ for $0 < m < m_0(c+1)$. It is easy to verify that the point where $F'(c,m) = 0$ does not give a local minimum for R.

We conclude this section with a simple remark on the asymptotic properties of $R(c,M)$.

Corollary 2. Let $c \geqq c_0$ and let $M \to \infty$. Then

$$\lim R(c,M)/M = \lim D^-R(c,M) = \lim D^+R(c,M) \geqq d(c,m_0(c+1)),$$

where all limits exist and are finite.

Proof. The proof follows from the asymptotic theory of concave functions noting that the directions of recession of $R(c,M)$ are bounded by $d(c,m_0(c+1))$.

7. The minimal regret and the optimal sampling plan

In this section we shall show that $R(M) = \min_{c} R(c,M)$ is composed of successive segments of the functions $R(c,M)$ corresponding to increasing values of c.

From the theorem on the minimum of concave functions it follows immediately that $R(M)$ is concave and hence continuous and differentiable except for at most countably many values of M. However, to give a description of $R(M)$ in terms of the known functions we shall use a more constructive approach. Let

$$R^{\circ}(c,M) = \min_{i \leq c} R(i,M) \quad \text{for} \quad c = 0,1,2,\ldots \tag{57}$$

For fixed M it is obvious that $R^{\circ}(c,M)$ is non-increasing with c. Moreover we have $R^{\circ}(c,M) \to R(M)$ for $c \to \infty$. The following shows how R° may be constructed.

Lemma 3. Let

$$R(c+1,M) < R^{\circ}(c,M) \tag{58}$$

for some M. Then (58) holds for an open half-line $M_{c+0.5} < M < \infty$, say. Moreover, for any version of $m(c+1,M)$ we have $m(c+1,M) > m_0(c+1)$ for $M > M_{c+0.5}$.

Proof. Assume that (58) holds for $M = M_1$ and let $m_1 = m(c+1,M_1)$ for some version of $m(c+1,M)$. It is easily verified that $m_1 > 0$. From (58) we find

$$R(c+1,m_1,M_1) < R^{\circ}(c,M_1) \leqq R(c,m_1,M_1)$$

which according to (42) leads to $\Delta d(c,m_1) < 0$. It then follows from Theorem 1 that $c < c_0(m_1)$ or, equivalently, that $m_0(c+1) < m_1$ such that

$$d(c+1,m_1) < d(c+1,m_0(c+1)) = d(c,m_0(c+1)).$$

Moreover, Corrollary 2 to Theorem 3 implies that $D^+R^{\circ}(c,M) \geqq d(c,m_0(c+1))$ which combined with the result above gives

$$D^+R^{\circ}(c,M) > d(c+1,m_1) = d(c+1,m(c+1,M_1)).$$

Using the fact that $m(c+1,M)$ is non-decreasing, see Theorem 3, and that $d(c+1,m)$ is decreasing for $m < m_0(c+2)$ we get

$$D^+R^{\circ}(c,M) > d(c+1,m(c+1,M)) \geqq D^+R(c+1,M) \quad \text{for} \quad M_1 < M.$$

Hence $D^+(R^{\circ}(c,M) - R(c+1,M)) > 0$ for $M_1 < M$ so that

$$R^{\circ}(c,M) - R(c+1,M) > R^{\circ}(c,M_1) - R(c+1,M_1) > 0.$$

The proof is completed observing that we may put

$$M_{c+0.5} = \inf\{M: R(c+1,M) < R^\circ(c,M)\}.$$

In view of the lemma we may extend the discussion in Section 6 to cover the minimal regret, $R(M)$.

Theorem 4. $R(M)$ is an increasing and concave function of M. The equation $R(c,m,M) = R(M)$ admits a solution $c = c_M, m = m(c_M,M)$, where c_M and $m(c_M,M)$ are non-decreasing and continuous except for at most countably many values of M. The one-sided derivatives of $R(M)$ exist and are non-increasing functions of M satisfying $D^-R(M) \geqq d(c_M, m(c_M,M)) \geqq D^+R(M)$ with equality signs holding except for at most countably many values of M.

Proof. Successive applications of Lemma 3 for $c = 0,1,2,\ldots$ show that $R^\circ(c,M)$ is composed of consecutive parts of $R(i,M)$, $i = 0,1,2,\ldots,c$. The theorem then follows from Theorem 3.

It is obvious that c_M is not uniquely defined in its discontinuity points. However, since the minimal regret is a continuous function of M we are free to choose any version of c_M without affecting the regret.

We note that Lemma 3 implies $m_0(c_M) \leqq m(c_M,M)$. It follows that

$$\delta m_0(c_M) \leqq R(M), \tag{59}$$

and

$$c_M \leqq c_0(R(M)/\delta) \tag{60}$$

which gives an upper bound for the optimal acceptance number corresponding to any given lot size. Thus, to obtain the optimal sampling plan we need only consider a limited number of c-values. If, e.g. we want to determine $R(M)$ for $M < M_0$, we may successively determine $R^\circ(c,M)$ for $M < M_0$ and $c = 0,1,2,\ldots$ and stop the process when the condition

$$\delta m_0(c) > R^\circ(c,M_0) \tag{61}$$

is met.

A typical graph of $R(M)$ has been given in Fig. 8.

8. The tabular solution

Theorem 4 leads in a natural way to a rough tabulation of the optimal plan $(c_M, m(c_M,M))$ as a function of M in the cases where $m(c_M,M)$ has only finitely many discontinuities in any finite M-interval.

Since $m(c_M,M)$ is a non-decreasing function of M, the discontinuity points of $m(c_M,M)$ will in most cases give a sufficiently accurate description of the optimal sample size.

Defining $R^* = (K - Mk_0)/(k_s(\bar{\lambda}) - k_0)$, where k_0 is defined corresponding to λ_r, we find

$$R^*(c,m,M) = m + (M - m)\{\gamma_1\, Q(\lambda_1) + \gamma_2\, P(\gamma_2)\} \tag{63}$$

where

$$\left.\begin{aligned} \gamma_1 = w_1(\lambda_r - \lambda_1)/(\lambda_s - \lambda_0),\quad \gamma_2 = w_2(\lambda_2 - \lambda_r)/(\lambda_s - \lambda_0), \\ \lambda_1 < \lambda_r < \lambda_2,\ \lambda_0 = w_1\lambda_1 + w_2\lambda_r \text{ and } \lambda_s = \lambda_r + \{k_s(\bar{\lambda}) - k_r(\bar{\lambda})\}/(A_2 - R_2). \end{aligned}\right\} \tag{64}$$

In (63) we have still four parameters left, viz. $(\lambda_1, \lambda_2, \gamma_1, \gamma_2)$. We may, however, eliminate one by proper choice of observational unit. Hence, we shall assume that $\lambda_1 = 1$ (instead of $\lambda_r = 1$ as in Section 3), and we shall write $r = \lambda_2/\lambda_1$, which means that

$$R^*(c,m,M) = m + (M - m)\{\gamma_1(1 - B(c,m)) + \gamma_2 B(c,rm)\}. \tag{65}$$

This form shows that for the two-point prior the coefficients (γ_1, γ_2) may be interpreted as the product of a prior probability and a decision loss and that this loss need not be derived from linear cost functions but may be chosen arbitrarily. In this wider context the parameter λ_r does not have any direct interpretation and hence we have chosen to use λ_1 as "reference quality level" instead of λ_r.

We note that R^* has the same structure as (26) for $\delta = 1$ and $d(c,m) = \gamma_1(1 - B(c,m)) + \gamma_2 B(c,rm)$, the difference being a scale transformation only.

A table of the optimum relationship between c, m and M depending on the three parameters γ_1, γ_2 and r would be rather extensive. It is, however, possible to get a good approximation from a table depending on only two parameters, since *the optimum plan corresponding to* $(M, \gamma_1, \gamma_2, r)$ *is approximately equal to the optimum plan for* $(M\gamma_1, 1, \gamma_2/\gamma_1, r)$. Therefore, optimum plans have been tabulated by minimizing R^* for $\gamma_1 = 1$ only, see the table in the Appendix.

The table contains optimum plans for $\gamma_1 = 1$, eight values of γ_2 between 0.04 and 1.00, and twelve values of r between 2 and 25, with the double limitation that $M < 10{,}000$ and $c \leqq 19$.

To derive the results stated above we introduce $M^* = M\gamma_1$ and $\gamma_2^* = \gamma_2/\gamma_1$ into (65) so that

$$R^*(c,m,M) = m + (M^* - m)\{(1 - B(c,m)) + \gamma_2^*\, B(c,rm)\} + E, \tag{66}$$

where

$$E = m(1 - \gamma_1)\{(1 - B(c,m)) + \gamma_2^*\, B(c,rm)\}.$$

It will be proved later on that the remainder term E is of smaller order of magnitude than the main term for $M \to \infty$ which means that an approximation to the optimum plan may be found from a table with $\gamma_1 = 1$ using M^* and γ_2^* as arguments.

A great number of comparisons of the exact and the approximate solution has been carried out with the result that the approximation obtained is satisfactory for practical purpose for $\gamma_1 > 0.2$ and $r > 2$.

As an example consider the problem of finding the optimum plans for $\gamma_1 = 0.8$, $\gamma_2 = 0.2$ and $r = 3$. The exact solution has been given in Table 1 for $c \leqq 10$, and

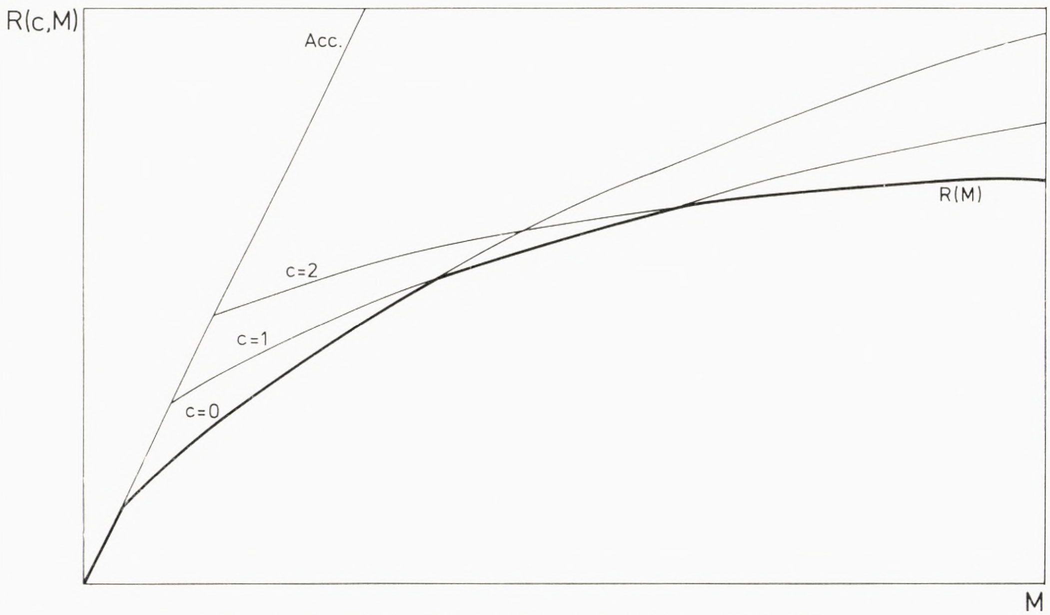

Fig. 8. $R(c,M)$ for three values of c. $R(M) = \inf R(c,M)$.

For the two prior distributions previously considered the tabulation along these lines is fairly simple because $d(c,m)$ satisfies the conditions of Corollary 1. The discontinuities in the optimal sample size will then correspond to the shifts of the optimal acceptance numbers.

The tables in the Appendix give the values of M corresponding to shifts in c_M, starting in most cases with the greatest lot size for which acceptance without inspection is to be preferred. Each line in the table gives an interval for M, the optimal acceptance number for this interval and the lower and upper limit, m_{cl} and m_{cu}, say, for the optimal sample size. However, since the tables have been organized such that the M-intervals are consecutive only the upper limit, $M_{c+0.5}$ say, of each interval is given. The tabulated lot and sample sizes have been given with three significant figures and at most four decimal places.

9. Optimal sampling plans for the two-point prior distribution

Since the Poisson solution may be used as approximation to the binomial solution, which has been studied in a previous paper (Hald, 1965), we shall formulate the results so that they are easily comparable to the binomial ones.

To derive a convenient form of the Poisson model for the two-point prior distribution from the assumptions in Section 3 we define the break-even quality as

$$\lambda_r = (R_1 - A_1)/(A_2 - R_2). \tag{62}$$

TABLE 1.

Comparison of exact solution for $\gamma_1 = 0.8$, $\gamma_2 = 0.2$, and $r = 3$ and approximate solution derived from the exact solution for $\gamma_1 = 1$, $\gamma_2 = 0.25$ and r = 3.

	Exact			Approximation		
c	$M_{c+0.5}$	m_{cl}	m_{cu}	$M_{c+0.5}$	m_{cl}	m_{cu}
	13.8	Accept		13.1	Accept	
2	16.9	0.702	0.756	16.5	0.696	0.758
3	22.2	1.22	1.29	21.9	1.21	1.29
4	28.7	1.76	1.83	28.5	1.76	1.83
5	36.4	2.31	2.38	36.3	2.31	2.38
6	45.5	2.86	2.92	45.6	2.86	2.92
7	56.3	3.41	3.47	56.5	3.41	3.47
8	69.1	3.96	4.01	69.5	3.96	4.01
9	84.3	4.51	4.56	84.8	4.51	4.56
10	102	5.06	5.11	103	5.06	5.11

this has been compared with the approximate solution derived from the table in the Appendix for $\gamma_1 = 1$ and $\gamma_2 = 0.2/0.8 = 0.25$ by dividing the tabulated values of M by 0.8. It will be seen that the approximation is very good.

To derive asymptotic expansions of the optimum (c,m) in terms of M for $M \to \infty$ we shall consider (c,m) as positive real variables and define $B(c,m)$ by means of the incomplete Gamma-function, i.e.

$$B(c,m) = \frac{1}{\Gamma(c+1)} \int_m^\infty x^c e^{-x}\, dx.$$

By a similar procedure as in a previous paper, see HALD (1967), we find for $m \to \infty$ and $c/m = h$, where h is a positive constant, that

$$B(c,m) = b(c,m)\frac{1}{|1-h|}\left\{1 - \frac{h}{m(1-h)^2} + O(m^{-2})\right\} \quad \text{for} \quad \frac{c}{m} = h < 1,$$

the same expression being valid for $1 - B(c,m)$ when $h > 1$. By means of Stirling's formula for $\Gamma(c+1)$ and the above result we get an asymptotic expansion for the Producer's and the Consumer's risk.

Introducing this expansion into R^* and minimizing with respect to $h = c/m$ and m we get an expansion analogous to the one for the binomial case, see HALD (1965, 1967 a). The expansion is shown in the theorem below. The same result may naturally also be obtained starting from the binomial case and using the standard approximation of the binomial distribution by the Poisson distribution.

Theorem 5. For the Bayesian single sampling plan corresponding to a two-point prior distribution we have for $M \to \infty$ that

$$c = md_0 + d_2 + d_4 m^{-1} + O(m^{-2}), \tag{67}$$

where

$$d_0 \ln r = r - 1,$$

$$d_2 \ln r = \ln \left\{ \frac{\gamma_1 (r - d_0) \ln d_0}{\gamma_2 r (d_0 - 1) \ln (r/d_0)} \right\}$$

and

$$d_4 \ln r = \frac{d_0}{(r-d_0)^2} - \frac{d_0}{(d_0-1)^2} - \frac{d_2}{r-d_0} - \frac{d_2}{d_0-1}.$$

Further

$$\min_{(c,\,m)} R^*(c, m, M) = m + \frac{1}{e_0}\left(1 - \frac{1}{2\,e_0 m}\right) + O(m^{-2}), \tag{68}$$

where m is determined from

$$\ln(M-m) = e_0 m + \tfrac{1}{2}\ln m + e_2 - \ln\left\{\frac{\gamma_1 e_0 \ln r}{\sqrt{(2\pi d_0)}(d_0-1)\ln(r/d_0)}\right\} + \left(e_4 - \frac{1}{2e_0}\right)m^{-1} + O(m^{-2}), \tag{69}$$

and

$$e_0 = d_0 \ln d_0 + 1 - d_0,$$

$$e_2 = d_2 \ln d_0$$

and

$$e_4 = d_4 \ln d_0 + \frac{d_2 + d_2^2}{2d_0} + \frac{1}{12 d_0} + \frac{d_2}{d_0 - 1} + \frac{d_0}{(d_0-1)^2}.$$

The two risks for the optimum plan are

$$1 - B(c, m) = \frac{\ln(r/d_0)}{\gamma_1 e_0 \ln r}\left\{1 - \frac{1}{2\,e_0 m} + O(m^{-2})\right\}\frac{1}{M-m} \tag{70}$$

and

$$B(c, rm) = \frac{\ln d_0}{\gamma_2 e_0 \ln r}\left\{1 - \frac{1}{2\,e_0 m} + O(m^{-2})\right\}\frac{1}{M-m}. \tag{71}$$

It follows from this theorem that the optimum m asymptotically is proportional to $\ln M$ and that c approximately is a linear function of m. The minimum regret consists essentially of two terms, the sampling costs, m, and the average decision loss, which tends to a constant, $1/e_0$. Thus, for large lots the sampling costs will dominate over the decision loss. The two risks tend to zero proportional to $1/M$.

The main terms of (69) may be written as

$$\ln(M\gamma_1) = e_0 m + \tfrac{1}{2}\ln m + \lambda + \left(\ln\frac{\gamma_1}{\gamma_2}\right)\frac{\ln d_0}{\ln r} + O(m^{-1}),$$

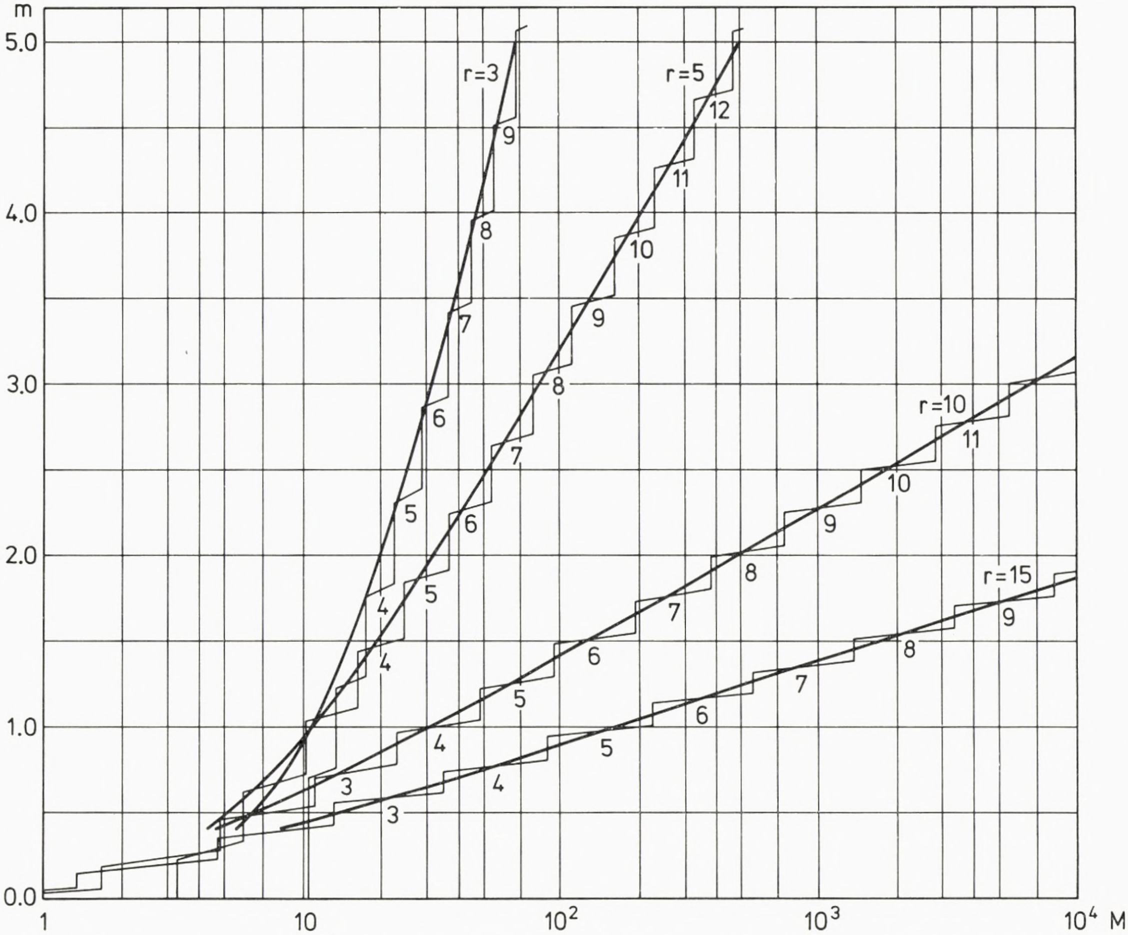

Fig. 9. The exact and the asymptotic sampling plan as function of M for $\gamma_1 = 1$ and $\gamma_2 = 0.25$. The asymptotic plans have been plotted for $m > 0.4$ only. The numbers indicated below the steps of the curve are values of c_M.

where λ is a constant independent of (γ_1, γ_2). Since e_0 and d_0 are functions of r only it follows that asymptotically m depends on $M^* = M\gamma_1$ and $\gamma_2^* = \gamma_2/\gamma_1$. As shown in the example above this result also holds with good approximation for small values of M.

To facilitate the use of the asymptotic formulas the most important coefficients have been tabulated as functions of r for $\gamma_1 = \gamma_2 = 1$, see the tables in the Appendix. A typical picture of the relation between the exact and the asymptotic solution has been given in Fig. 9.

The asymptotic formulas serve several purposes.

(1) Starting from c we may determine the corresponding M-interval and within that the relation between m and M.

(2) Starting from M we may determine the corresponding m, and from m determine c.

(3) They may be used for developing interpolation formulas.

The first method is useful for making a systematic tabulation of sampling plans whereas the second is suitable for computing "isolated" plans for a given M.

Starting from an integer value of c we first find $m(c)$, say, from (67) and the corresponding $M(c)$ from (69). Similarly we find $M(c-0.5)$ and $M(c+0.5)$, being the approximations to the lower and upper limit for M having c as optimum acceptance number.

In the asymptotic solution we have disregarded the discreteness of c. By investigation of the relationship between m and M for given (integer) value of c we find that m is approximately a linear function of $\ln M$ with slope $1/r$ so that

$$m \simeq m(c) + r^{-1} \ln\{M/M(c)\} \quad \text{for} \quad M(c-0.5) < M < M(c+0.5), \tag{72}$$

which for small intervals may be replaced by

$$m \simeq m(c) + \{M - M(c)\}/\{rM(c)\} \text{ for } M(c-0.5) < M < M(c+0.5). \tag{73}$$

Since m_{cu} corresponds to $M_{c+0.5}$ we find

$$m_{cu} \simeq m(c) + r^{-1} \ln\{M(c+0.5)/M(c)\},$$

which by means of (67) and (69) leads to the following approximation

$$m_{cu} \simeq m(c) + \frac{1}{2rd_0}\left(e_0 + \frac{1}{2\,m(c)}\right). \tag{74}$$

A similar argument shows that m_{cl} is obtained by subtraction of the correction term in (74).

The approximation obtained by using (67), (69), and (72) is usually very good even for quite small values of c. *Normally the approximate value of c will deviate at most* $\pm$ 1 *from the correct value which secures a high efficiency of the approximate plan since the relation between c and m is very accurate*, see the discussion on efficiency in previous papers.

The approximation depends on the value of r, being good for large values of r and poorer for small values. Two typical examples have been shown in Table 2.

The approximation may break down for values of M for which the cheapest solution is acceptance without inspection since the formula may lead to a sampling plan even if no optimum plan exists. The difference in costs by using such a plan instead of accepting without inspection will, however, usually be small.

If one wants to be sure to find the optimum plan one may start from the approximation and compare the costs of this plan with the costs of suitably chosen neighbouring plans and with $M\gamma_2$ thus finding the optimum procedure by trial and error.

To determine the sampling plan corresponding to a given lot size we shall use the inverse formula giving m as function of M which may be found as in the previous paper, see Hald (1965).

TABLE 2.

Comparison of exact and approximate sampling plans computed from the asymptotic formulas.

$\gamma_1 = 1,\ \gamma_2 = 0.25,\ r = 3.$ $m(c) = 0.5493c - 0.4162$ $m_{cu} = m(c) + 0.0247 + 0.0458/m(c)$ $\ln(M - m) = 0.2702m + \frac{1}{2}\ln m + 1.9951$	$\gamma_1 = 1,\ \gamma_2 = 0.25,\ r = 10.$ $m(c) = 0.2558c - 0.0217$ $m_{cu} = m(c) + 0.0310 + 0.00640/m(c)$ $\ln(M - m) = 2.4196m + \frac{1}{2}\ln m + 1.0037$

c	Approximation $M(c+0.5)$	Approximation m	Exact M	Exact m	Approximation $M(c+0.5)$	Approximation m	Exact M	Exact m
1	5.65	–	10.5	Accept	4.31	0.176–0.193	4.74	0.181–0.283
2	10.3	0.591–0.774	13.2	0.696–0.758	10.2	0.446–0.534	10.9	0.448–0.530
3	15.1	1.17–1.29	17.5	1.21–1.29	22.0	0.706–0.785	23.2	0.708–0.782
4	20.4	1.73–1.83	22.8	1.76–1.83	45.7	0.964–1.04	47.8	0.965–1.04
5	26.6	2.29–2.37	29.0	2.31–2.38	93.1	1.22–1.29	96.5	1.22–1.29
6	33.8	2.84–2.92	36.5	2.86–2.92	187	1.48–1.55	193	1.48–1.54
8	52.1	3.94–4.01	55.6	3.96–4.01	741	1.99–2.06	749	1.99–2.05
10	77.6	5.04–5.11	82.2	5.06–5.11	2820	2.50–2.57	2850	2.50–2.57

Introducing the auxiliary functions

$$g(M) = 1 - (2\ln M)^{-1} \tag{75}$$

and

$$f(M) = \ln M - \tfrac{1}{2}(\ln\ln M)\, g(M) \tag{76}$$

we get

$$e_0 m = f(M) - \left\{h - \tfrac{1}{2}\ln e_0 - \ln\gamma_1 + \left(\ln\frac{\gamma_1}{\gamma_2}\right)\frac{\ln d_0}{\ln r}\right\} g(M), \tag{77}$$

where $h - \frac{1}{2}\ln e_0$ has been tabulated in the Appendix.

From M we may find $m(M)$ by means of (77) and then $c(M) = m(M)d_0 + d_2$ from (67). Rounding to the nearest integer we may find the corresponding $m(c)$ and use (73) to obtain a corrected value of m.

Table 3 shows that (77) leads to good results for the two cases discussed in Table 2. The approximate value of c does not deviate more than 1 from the correct value. Many other cases have been investigated with the result that the deviations found are less than ± 1 for $r \geqq 3$.

As a final consequence of the theorem we shall give some rules for interpolation in the tables.

From (67) and (69) it follows that $\ln M_{c+0.5}$ *and* m_c *are approximately linear functions of* $\ln\gamma_2$ and, consequently, linear interpolation on $\log M$ and on m using γ_2 as argument is recommended.

Considering $M_{c+0.5}$ as function of r it will be found that it is first decreasing and then increasing. Within the tabulated range, $2 \leqq r \leqq 25$, we may, however, have only one of these branches. *In the neighbourhood of the minimum quadratic interpolation on* $\log M$ *should be used, otherwise linear interpolation on* $\log M$ *will normally suffice.*

TABLE 3.

Comparison of exact and approximate sampling plans computed from the asymptotic formulas.

	$\gamma_1 = 1,\ \gamma_2 = 0.25,\ r = 3.$ $m = 3.7014\{f(M) - 2.6495g(M)\}$ $c = 1.821m + 0.758$			$\gamma_1 = 1,\ \gamma_2 = 0.25,\ r = 10.$ $m = 0.41329\{f(M) - 0.56186g(M)\}$ $c = 3.909m + 0.085$		
	Approximation		Exact	Approximation		Exact
M	m	c	c	m	c	c
3	< 0	Acc.	Acc.	0.317	1	1
5	< 0	,,	,,	0.437	2	2
7	< 0	,,	,,	0.529	2	2
10	< 0	Acc.	Acc.	0.635	3	2
20	1.23	3	4	0.856	3	3
30	2.29	5	6	0.992	4	4
50	3.72	8	8	1.17	5	5
70	4.71	9	10	1.29	5	5
100	5.78	11	12	1.42	6	6

From (67) we obtain

$$m(c) = \left\{c - d_2^* - \left(\ln\frac{\gamma_1}{\gamma_2}\right)(\ln r)^{-1}\right\} d_0^{-1}.$$

(d_2^* equals d_2 for $\gamma_1 = \gamma_2$).

Since $-d_2^*$ is nearly constant and equal to 0.5 we find the following differences with respect to r

$$\Delta_r m(c) \simeq (c + 0.5)\Delta d_0^{-1} - \left(\ln\frac{\gamma_1}{\gamma_2}\right)\Delta(r-1)^{-1}.$$

As $\{\Delta(r-1)^{-1}\}/\{\Delta d_0^{-1}\} \to 0$ for $r \to \infty$ it follows that $m(c)$ for large r will be approximately linear in d_0^{-1}.

For *small* r the spacing of the tabular values is such that *linear interpolation on* $m(c)$ *with respect to* r *is sufficiently accurate*, whereas *for large* r *linear interpolation on* $m(c)$ *should be with* d_0^{-1} *as argument*. Numerical examples may be found in the Appendix.

We shall conclude this section with a discussion of the Poisson sampling plan as approximation to the binomial one.

The fundamental parameters in the binomial model are

p_r = the break-even quality expressed as a fraction defective, i.e. the cost of rejection per item divided by the cost of accepting a defective item,

p_s = the cost of sampling inspection per item divided by the cost of accepting a defective item,

p_1 = the process average fraction defective for good lots,

p_2 = the process average fraction defective for bad lots, and

w_2 = the proportion (or a priori probability) of bad lots, $w_1 + w_2 = 1$.

From these parameters we derive

$$\gamma_1 = w_1(p_r - p_1)/(p_s - p_0), \ \gamma_2 = w_2(p_2 - p_r)/(p_s - p_0) \tag{78}$$

where $p_0 = w_1 p_1 + w_2 p_r$, $p_1 < p_r < p_2$ and $p_r \leqq p_s$.

From the general form of the average regret, given by (3), we get

$$R_b(c,n,N) = (p_s - p_0)n + (N-n)\{(p_r - p_1)\, w_1 Q_b(p_1) + (p_2 - p_r) w_2 P_b(p_2)\},$$

where Q_b and P_b denote binomial error probabilities.

The "natural" parameters of the model are (p_1, p_2, w_2), which characterize the prior distribution, and (p_r, p_s), which depend on the costs. However, as in the Poisson case, we may reduce the number of parameters. Introducing $R_b^* = R_b/(p_s - p_0)$ we get

$$R_b^*(c,n,N) = n + (N-n)\{\gamma_1 Q_b(p_1) + \gamma_2 P_b(p_2)\}, \tag{79}$$

which shows that the solution depends on four parameters only, viz. $(p_1, p_2, \gamma_1, \gamma_2)$.

For $m = np_1$ and $p_2 = rp_1$ the usual Poisson approximation to the binomial probabilities leads to

$$R_b^*(c,n,N) \simeq n + (N-n)\{\gamma_1 Q(m) + \gamma_2 P(rm)\},$$

such that we may use R^* as an approximation to $p_1 R_b^*$ for small p_1 and p_2, $M = Np_1$. Thus, we shall use the Poisson solution as approximation to the binomial solution.

It is obvious that the accuracy of the approximation depends essentially on the size of the p's and on r. A large number of comparisons has been carried out for $0.01 \leqq p_r \leqq 0.10$, $r > 3$, and $100 \leqq N \leqq 100{,}000$. Some typical examples regarding the value of c have been shown in Table 4. In most cases the Poisson solution leads to the same value of c as the binomial, in some cases we get a value which is one unit too large, and particularly for $p_r = 0.10$, small values of r and large values of N we get a value of c which is two units larger than the binomial.

The relation between c and n is very accurate in nearly all cases.

The *general conclusion* is that for $p_r < 0.05$, $r < 3$, and $N < 100{,}000$ the acceptance number determined from the Poisson model will normally equal the corresponding binomial acceptance number and occasionally the Poisson solution will be 1 larger than the binomial. Even for p_r as large as 0.10 the Poisson approximation will be good for $r > 5$.

The tables in the Appendix of the present paper may therefore be regarded as a valuable supplement to the tables in the previous paper, see HALD (1965). It should be noted that the present tables usually will be easier to use because conversion formulas and interpolation are simpler here.

The prior distribution should be chosen such that it describes the expected quality variation for the lots in question. All kinds of information should be used in determining the prior distribution such as knowledge of normal quality variation in

TABLE 4.

Comparison of values of c found from the binomial and the Poisson model. The value tabulated is the binomial solution. The subscript indicates the Poisson solution minus the binomial. No subscript means that the two solutions are identical.

$100p_r$	1.0	1.0	1.0	2.0	3.0	1.0	3.0	10.0	10.0	10.0
$100p_s$	1.0	1.0	2.0	2.0	6.0	1.0	3.0	10.0	10.0	10.0
$100p_1$	0.6	0.6	0.6	0.6	1.8	0.6	1.8	2.0	4.0	6.0
$100p_2$	4.0	4.0	4.0	4.0	12.0	2.0	6.0	15.0	15.0	15.0
$100w_2$	5.0	10.0	5.0	5.0	10.0	5.0	5.0	5.0	5.0	5.0
γ_1	1	1	0.275	1	0.265	1	1	1	1	1
γ_2	0.395	0.833	0.109	0.0752	0.221	0.132	0.132	0.0329	0.0439	0.0658
r	6.67	6.67	6.67	6.67	6.67	3.33	3.33	7.50	3.75	2.50
N	c	c	c	c	c	c	c	c	c	c
100	0	0	Acc.	Acc.	0	Acc.	Acc.	Acc.	Acc.	Acc.
200	0	0	,,	,,	0	,,	,,	,,	,,	,,
300	1	1	Acc.	,,	0	,,	,,	,,	,,	,,
500	1	1	0	,,	1	,,	,,	,,	,,	,,
700	1	2	0	,,	1	,,	,,	,,	,,	,,
1,000	2	2	0	,,	2	,,	Acc.	Acc.	Acc.	,,
2,000	3	4	1	Acc.	3	,,	5	3	$5_{\text{Acc.}}$	Acc.
3,000	4	4	2	2	4	Acc.	6_1	4	7	11
5,000	5	5	2_1	3	5	4	9	5	9	15_1
7,000	5	6	3	4	5_1	5	10	5	10_1	18_1
10,000	6	7	4	5	6	7	12	6	11_1	20_2
20,000	7_1	8	5	6	7_1	10	15	7	14_1	26_2
30,000	8	9	6	7	8	12	17	8	16_1	29_2
50,000	9	10	7	8	9_1	14_1	19_1	9	18_1	33_2
70,000	10	11	7_1	9	10	16	21_1	9_1	19_1	36_2

the market and for the specific supplier, information from the supplier on recent changes of his production process including changes in the quality of raw materials, and observations on quality variations from past inspection.

Let us suppose that k lots have been inspected previously, either by sampling or total inspection, and that we want to estimate the parameters of the prior distribution from these k observations.

We shall first discuss the case of a double binomial distribution, i.e. $x_1, \ldots, x_k$ represent the number of defective items found in k samples (or lots) of size n, the distribution of x being

$$b_{2b}(x,n) = w_1 b(x,n,p_1) + w_2 b(x,n,p_2),$$

where $0 < p_1 < p_2 < 1$ and $0 < w_1 < 1$, $w_1 + w_2 = 1$.

Various estimation procedures have been discussed by BLISCHKE (1962, 1965), the simplest being the method of moments which we shall refer here. It consists in expressing the first three factorial moments by the three parameters (p_1, p_2, w_1), solving with respect to the parameters and using as estimates the resulting expressions with the population moments replaced by the sample moments.

Since
$$E\{x^{(\nu)}\} = n^{(\nu)} E\{p^\nu\} = n^{(\nu)}(w_1 p_1^\nu + w_2 p_2^\nu)$$
we define
$$\alpha_\nu = w_1 p_1^\nu + w_2 p_2^\nu$$
and
$$a_\nu = (kn^{(\nu)})^{-1} \sum_{i=1}^{k} x_i^{(\nu)},$$
where $E\{a_\nu\} = \alpha_\nu$. Introducing the auxiliary quantity
$$\beta = (\alpha_3 - \alpha_1\alpha_2)/(\alpha_2 - \alpha_1^2)$$
we find
$$\left.\begin{matrix} p_1 \\ p_2 \end{matrix}\right\} = \frac{\beta}{2} \mp \tfrac{1}{2}(\beta^2 - 4\beta\alpha_1 + 4\alpha_2)^{1/2} \tag{80}$$
and
$$w_1 = (p_2 - \alpha_1)/(p_2 - p_1).$$

Replacing α_ν by a_ν we get the estimates.

For the Poisson case $x_1, \ldots, x_k$ represent the number of defects found in k samples (or lots) of size m, the distribution of x being
$$b_2(x,m) = w_1 b(x, m\lambda_1) + w_2 b(x, m\lambda_2),$$
where $0 < \lambda_1 < \lambda_2$, $m > 0$, $0 < w_1 < 1$, and $w_1 + w_2 = 1$.

Since
$$E\{x^{(\nu)}\} = m^\nu E\{\lambda^\nu\} = m^\nu(w_1 \lambda_1^\nu + w_2 \lambda_\nu^2)$$
we define
$$\alpha_\nu = w_1 \lambda_1^\nu + w_2 \lambda_2^\nu$$
and
$$a_\nu = (km^\nu)^{-1} \sum_{i=1}^{k} x_i^{(\nu)},$$
where $E\{a_\nu\} = \alpha_\nu$. The solution with respect to $(\lambda_1, \lambda_2, w_1)$ is then the same as the one given above for (p_1, p_2, w_1).

10. Optimal sampling plans for the gamma prior distribution

The gamma prior (14) depends on the parameters s and τ. Since the break-even quality is assumed to be 1, it follows from Section 3 that the optimum plan depends on the four parameters (M, λ_s, s, τ). Instead of τ we shall use the mean of the prior distribution, $\bar{\lambda} = s/\tau$, because $\bar{\lambda}$ is easier to interpret. From $V\{\lambda\} = s/\tau^2 = \bar{\lambda}^2/s$ we get $s = (\bar{\lambda}/\sigma_\lambda)^2$ which shows that s^{-1} is the square of the coefficient of variation in the prior distribution.

The rejectable part of the prior distribution is

$$w_2 = \int_1^\infty w(\lambda)\,d\lambda = \int_\tau^\infty z^{s-1}e^{-z}\,dz/\Gamma(s). \tag{81}$$

A table of w_2 for suitably chosen values of s and $\bar{\lambda}$ has been given in the Appendix.

The case $\lambda_s = 1$, i.e. the case of rectifying inspection, see (24), is particularly important. Setting $\delta = \lambda_s - \lambda_0$ and $\delta_0 = 1 - \lambda_0$ we get

$$\left.\begin{aligned} R &= m\delta + (M - m)\,d(c, m) \\ &= \delta\delta_0^{-1}\{m\delta_0 + (M^* - m)d(c, m)\} + (\delta - \delta_0)\delta_0^{-1}md(c, m) \end{aligned}\right\} \tag{82}$$

where $M^* = M\delta_0/\delta$. It will be shown later on that $md(c,m) = O(1)$ for $M \to \infty$ whereas the first term is $O(\sqrt{M})$. Disregarding the last term it follows that *the optimum plan corresponding to* $(M,\lambda_s,s,\bar{\lambda})$ *is approximately equal to the optimum plan for* $(M^*,1,s,\bar{\lambda})$, *where* $M^* = M\delta_0/\delta$. Therefore, optimum plans have been tabulated for $\lambda_s = 1$ only, see the table in the Appendix, and auxiliary tables of δ_0 and δ_0/δ for different value of λ_s have been provided.

The Appendix contains tables of optimum plans for twelve values of s between 0.1 and 20 and thirteen values of $\bar{\lambda}$ between 0 and 1, with the double limitation that $M < 10{,}000$ and $c \leqq 19$. A great number of comparisons of the exact and the approximate solution for $\lambda_s > 1$ has been carried out leading to the conclusion that the approximation obtained is satisfactory for practical purposes for $\lambda_s < 5$ and $s < 10$.

As an example consider the problem of finding the optimum plans for $\lambda_s = 1.5$, $\bar{\lambda} = 0.5$ and $s = 0.3$. From the auxiliary table in the Appendix we read $\delta_0/\delta = 0.5777$ such that the optimum plans may be found (approximately) by entering the table for $\lambda_s = 1$, $\bar{\lambda} = 0.5$ and $s = 0.3$ using $0.5777M$ as argument. The exact solution has been given in Table 5 for $c \leqq 10$, and this has been compared with the approximate solution derived from the table in the Appendix by dividing the tabulated values of M by 0.5777. It will be seen that the approximation is very good.

To derive asymptotic expansions of the optimum (c,m) in terms of M for $M \to \infty$ we consider (c,m) as positive real variables and use the following

Lemma 4. Let

$$B_w(x, m) = \int_0^\infty B(x, m\lambda)\,w(\lambda)\,d\lambda,$$

where $w(\lambda)$ is three times continuously differentiable in the neighbourhood of $h = x/m$. For fixed h, $0 < h < \infty$, and $m \to \infty$ we have

TABLE 5.

Comparison of exact solution for $\lambda_s = 1.5$, $\bar{\lambda} = 0.5$ and $s = 0.3$ and approximate solution derived from the exact solution for $\lambda_s = 1$, $\bar{\lambda} = 0.5$ and $_s = 0.3$.

	Exact			Approximation		
c	$M_{c+0.5}$	m_{cl}	m_{cu}	$M_{c+0.5}$	m_{cl}	m_{cu}
	1.71	Accept		1.48	Accept	
0	23.0	0.0000	0.475	22.5	0.0000	0.478
1	76.6	0.986	1.38	76.2	0.979	1.39
2	160	2.06	2.34	160	2.06	2.34
3	274	3.10	3.31	273	3.10	3.31
4	418	4.12	4.30	417	4.12	4.30
5	592	5.13	5.28	592	5.13	5.28
6	796	6.14	6.27	795	6.14	6.27
7	1030	7.15	7.26	1030	7.15	7.27
8	1290	8.16	8.26	1290	8.16	8.26
9	1590	9.16	9.25	1590	9.16	9.25
10	1910	10.2	10.2	1900	10.2	10.2

$$B_w(mh, m) = \int_0^h w(\lambda)d\lambda + (w + \tfrac{1}{2}hw')m^{-1}$$
$$+ (w' + \tfrac{5}{6}hw'' + \tfrac{1}{8}h^2 w''')m^{-2} + O(m^{-3}),$$

where $w^{(\mu)} = w^{(\mu)}(h), \mu = 0,1,\ldots$.

If $w(\lambda)$ is a probability distribution $B_w(x,m)$ represents a mixed Poisson distribution. However, the lemma is valid whether $w(\lambda)$ is a density or not. The lemma may be proved analogously to the corresponding lemma for the mixed binomial (HALD, 1968a) or it may be found from this lemma by introducing the usual Poisson approximation to the binomial probabilities.

The lemma may be used to find an asymptotic expansion of $d(c,m)$, see (28), and thus an expansion of R. The optimum values of h and m are then obtained from the two equations $\partial R/\partial h = 0$ and $\partial R/\partial m = 0$, see the corresponding procedure for the binomial case in the previous paper (HALD, 1968b).

Theorem 7. For the Bayesian single sampling plan corresponding to a gamma prior distribution with parameters (s,τ) we have for $M \to \infty$ that

$$c = m + \tau - s - \tfrac{1}{2} + O(m^{-1}) \tag{83}$$

and

$$m = k_1\sqrt{M} + k_2 + O(M^{-1/2}), \tag{84}$$

where

$$k_1^2 = \tau^s e^{-\tau}/\{2\Gamma(s)\delta\} \tag{85}$$

and

$$12k_2 = -3(s-\tau)^2 + s - 10\tau - 1. \tag{86}$$

The minimum regret is

$$R(M) = (2m - k_1^2 - k_2)\delta + O(m^{-1}), \tag{87}$$

where m denotes the optimum sample size.

It follows from this theorem that the optimum m is approximately a linear function of $\sqrt{M}$ and that c is approximately a linear function of m. The minimum regret consists essentially of two terms, the sampling costs, δm, and the average decision loss, $\delta(m - k_1^2 - k_2)$. Thus, for large lots the sampling costs and the average decision loss will be approximately equal.

To facilitate the use of the asymptotic formulas the most important coefficients have been tabulated as functions of s and $\bar{\lambda} = s/\tau$, see the tables in the Appendix. It should be noted that these tables supplement the corresponding ones for the binomial case (Hald, 1968b) in the sense that they give limiting values for $p_r \to 0$.

Let us define the average probability of acceptance for lots which ought to be rejected, i.e. the Bayesian Consumer's risk, as

$$P_2 = \int_1^\infty B(c, m\lambda)\, w(\lambda) d\lambda / w_2,$$

where w_2 has been defined in (81). Similarly we define the Bayesian Producer's risk as

$$Q_1 = \int_0^1 \{1 - B(c, m\lambda)\} w(\lambda)\, d\lambda / w_1,$$

where $w_1 = 1 - w_2$. It may be proved that for the optimal plan these risks are of the same order of magnitude and tends to zero as $m^{-1/2}$, see Hald (1967b). Specifically

$$w_1 Q_1 = \{\delta k_1 w(1)/(\pi\sqrt{M})\}^{1/2}\{1 + O(M^{-1/4})\} = w_2 P_2 \{1 + O(M^{-1/4})\}.$$

The asymptotic formulas may be used for tabulating optimum plans. To each *integer* c we first find $m(c) = c - \tau + s + \frac{1}{2}$ from (83) and then $M(c)$ from $\sqrt{M(c)} = (m(c) - k_2)/k_1$, see (84). Investigating the relation between m and M for given integral c as in the paper on the analogous binomial problem (Hald, 1968b) we find that m is approximately a linear function of $\sqrt{M}$ within the interval $\sqrt{M(c)} \pm (2k_1)^{-1}$. The interval for M computed from the equations

$$\sqrt{M} = \sqrt{M(c)} \pm (2k_1)^{-1} \tag{88}$$

will have c as optimum acceptance number and the corresponding interval for m is found as

$$m = m(c) \pm \{2m(c+1)\}^{-1}. \tag{89}$$

Table 6.

Comparison of exact and approximate sampling plans computed from the asymptotic formulas.

	$s = 0.2,\ \bar{\lambda} = 0.9,\ \lambda_s = 1.\ \delta = 0.6588.$ $m = c + 0.4778,$ $m_{cu} = m(c) + 0.5/(m(c) + 1).$ $\sqrt{M(c+0.5)} = (m(c) + 0.7520)/0.3130.$				$s = 5,\ \bar{\lambda} = 0.9,\ \lambda_s = 1.\ \delta = 0.2186.$ $m = c - 0.0556,$ $m_{cu} = m(c) + 0.5/(m(c) + 1).$ $\sqrt{M(c+0.5)} = (m(c) + 4.873)/1.3965.$			
	Approximation		Exact		Approximation		Exact	
c	$M(c+0.5)$	m	$M_{c+0.5}$	m	$M(c+0.5)$	m	$M_{c+0.5}$	m
	0.539	–	0.0253	Accept			1.39	Accept
0	15.4	0.139–0.816	14.6	0–0.723	11.9	–	4.76	0–0.240
1	50.7	1.28–1.68	49.8	1.26–1.65	17.4	0.687–1.20	10.3	0.779–1.12
2	106	2.33–2.62	106	2.33–2.61	23.8	1.77–2.11	16.8	1.84–2.07
3	183	3.37–3.59	182	3.37–3.59	31.3	2.82–3.07	24.4	2.87–3.05
4	279	4.39–4.57	278	4.39–4.57	39.9	3.84–4.05	32.9	3.88–4.03
5	396	5.40–5.56	395	5.41–5.56	49.4	4.86–5.03	42.5	4.89–5.02
6	533	6.41–6.55	532	6.42–6.55	60.0	5.87–6.02	53.1	5.90–6.01
8	869	8.43–8.53	868	8.43–8.53	84.2	7.89–8.00	77.3	7.91–8.00
10	1290	10.4–10.5	1290	10.4–10.5	113	9.90–9.99	106	9.91–9.99
15	2690	15.4–15.5	2690	15.5–15.5	201	14.9–15.0	194	14.9–15.0
19	4180	19.4–19.5	4180	19.5–19.5	291	18.9–19.0	284	18.9–19.0

One may naturally also use the asymptotic formulas directly to find m and c corresponding to a given lot size M. However, the value of c found has to be rounded to the nearest integer and the optimum m corresponding to the given M and the integral c may then be found as described above from the equation

$$m - m(c) = \{\sqrt{M} - \sqrt{M(c)}\}k_1/m(c+1) \quad \text{for} \quad |\sqrt{M} - \sqrt{M(c)}| < (2k_1)^{-1}. \tag{90}$$

The discreteness of c will also influence the average decision loss. As an approximation we have

$$R(M) = \delta\{m + (m - k_1^2 - k_2)(1 - 2\varrho)^{-1}\}, \tag{91}$$

where $\varrho = m - m(c)$ is defined by (90). We remark that if $m(c)$ or $\sqrt{M(c)}$ is found to be negative the optimal plan is to accept without inspection.

The accuracy of the asymptotic formulas has been investigated numerically by comparing with the exact solution for a large number of cases. *Normally the approximate value of c will deviate at most 1 from the correct value.* Two examples, showing the results for a typical good and poor case, have been given in Table 6. The first entry for the approximate solution has been determined as $M(c - 0.5)$, where c is the first tabulated acceptance number.

Table 7 shows for the same two examples a comparison for selected values of M. The efficiency of the approximation, measured by the ratio of $R(M)$ and the value of $R(c,m,M)$ for the approximate plan, is larger than 0.86 for all values of M considered and in most cases the efficiency is much higher.

Table 7.

Comparison of exact and approximate sampling plans computed from the asymptotic formulas.

	$s = 0.2,\ \bar{\lambda} = 0.9,\ \lambda_s = 1.\ \delta = 0.6588.$ $m(M) = 0.3130\sqrt{M} - 0.2520,$ $c(M) = m(M) - 0.4778.$ $\sqrt{M(c)} = (c + 0.7298)/0.3130.$ $m = m(c) + 0.3130[\sqrt{M} - \sqrt{M(c)}]/(m(c)+1).$					$s = 5,\ \bar{\lambda} = 0.9,\ \lambda_s = 1.\ \delta = 0.2186.$ $m(M) = 1.3965\sqrt{M} - 4.373,$ $c(M) = m(M) + 0.0556.$ $\sqrt{M(c)} = (c + 4.318)/1.3965.$ $m = m(c) + 1.3965[\sqrt{M} - \sqrt{M(c)}]/(m(c)+1).$				
	Approximation		Exact		Efficiency	Approximation		Exact		Efficiency
M	c	m	c	m		c	m	c	m	
10	0	0.654	0	0.658	1.00	0	0.049	1	1.11	0.86
20	1	1.35	1	1.37	1.00	2	1.92	3	2.96	0.98
30	1	1.47	1	1.51	1.00	3	3.03	4	3.99	0.98
50	1	1.67	2	2.33	1.00	6	5.88	6	5.98	1.00
70	2	2.45	2	2.47	1.00	7	6.99	8	7.95	1.00
100	2	2.59	2	2.59	1.00	10	9.91	10	9.96	1.00
200	4	4.42	4	4.44	1.00	15	15.0	16	15.9	1.00
300	5	5.43	5	5.44	1.00					
500	6	6.51	6	6.52	1.00					
700	8	8.43	8	8.44	1.00					
1000	9	9.49	9	9.50	1.00					
2000	13	13.5	13	13.5	1.00					
3000	16	16.5	16	16.5	1.00					

Since the tables of the optimum plans have been organized with s and $\bar{\lambda}$ as parameters we shall formulate the interpolation rules in terms of these parameters.

It follows from Theorem 7 that $\sqrt{M_{c+0.5}}$ *and* m_c *are approximately linear functions of* $\bar{\lambda}^{-1}$ and, consequently, linear interpolation on $\sqrt{M}$ and on m using $\bar{\lambda}^{-1}$ as argument is recommended.

Considering $M_{c+0.5}$ as function of s it will be found that it is first decreasing and then increasing. Within a major part of the tabulated range, both branches are present. *In the neighbourhood of the minimum quadratic interpolation on* $\sqrt{M_{c+0.5}}$ *should be used, otherwise linear interpolation on* $\sqrt{M_{c+0.5}}$ *will normally suffice. In both cases use* log$s$ *as argument.* Finally m_{cl} and m_{cu} *are approximately linear functions of* s such that *linear interpolation on* m will be satisfactory.

The Poisson solution corresponding to a gamma prior distribution may be used as approximation to the binomial solution corresponding to a beta prior distribution which has been studied in a previous paper (Hald, 1968b).

The beta-binomial model is given by (3) for

$$dF(p) = p^{s-1}q^{t-1}dp/\beta(s,t),\ s > 0,\ t > 0,$$

$\beta(s,t)$ denoting the Beta-function, and $\delta_1 = p_s - p_0$, where

$$p_0 = \int_0^{p_r} p\,dF(p) + \int_{p_r}^{1} p_r dF(\mathrm{p}), \tag{92}$$

(p_s, p_r) being defined as in Section 9. The mean, $\bar{p}$ say, of the prior distribution is $\bar{p} = s/(s+t)$.

Setting $p = \lambda p_r$ and $\bar{p} = \bar{\lambda} p_r$ we find for fixed $(s, \bar{\lambda})$ and $p_r \to 0$ that the distribution of λ tends to

$$dW(\lambda) = (\tau\lambda)^{s-1} e^{-\tau\lambda} d(\tau\lambda)/\Gamma(s),$$

where $\tau = s/\bar{\lambda}$, i.e. λ has a gamma distribution with parameters (s, τ) and $E(\lambda) = \bar{\lambda} = s/\tau$.

Defining $\delta = \delta_1/p_r$, $M = Np_r$ and $m = np_r$ it follows that $R(c,m,M)$ for a gamma prior distribution may be obtained as the limit of $R_b(c,n,N)$ for a beta prior distribution for $p_r \to 0$ and $m \to \infty$, keeping $m = np_r$ fixed.

Since we have defined the variable λ as p/p_r it follows that the break-even quality expressed in the new scale equals 1.

It is obvious that the accuracy of the approximation depends essentially on the size of p_r. A large number of comparisons has been carried out for $0.01 \leqq p_r \leqq 0.10$, different combinations of $(s, \bar{p})$ and $100 \leqq N \leqq 100{,}000$. Some typical examples regarding the value of c have been shown in Table 8.

In most cases the Poisson solution leads to the same acceptance number as the binomial, and in some cases we find a value which is one unit too large.

For the region of parameters s and $\bar{\lambda} = \bar{p}/p_r$ for which tables have been given in the Appendix the *general conclusion* is that for $p_r < 0.1$ and $N < 100{,}000$ the acceptance number determined from the Poisson model will normally equal the corresponding binomial acceptance number, occasionally it will be one unit too large and in very few cases, corresponding to the largest value of s for given $\bar{\lambda}$, two units too large.

The tables in the Appendix may therefore be regarded as a valuable tool for the construction of binomial sampling plans, see HALD (1968b).

To estimate the parameters of the prior distribution from past observations we shall use the method of moments.

Let $x_1, \ldots, x_k$ represent the number of defective items found in k samples (or lots) of size n, the distribution of x being

$$b_{\beta b}(x,n) = \int_0^1 b(x,n,p)\,dF(p)$$

where

$$dF(p) = p^{s-1} q^{t-1} dp/\beta(s,t) \quad \text{for} \quad s > 0 \text{ and } t > 0.$$

Since

$$E\{p\} = \bar{p} = s/(s+t) \tag{93}$$

TABLE 8.

Comparison of values of $c \leqq 19$ found from the binomial and the Poisson model. The value tabulated is the binomial solution. The subscript indicates the Poisson solution minus the binomial. No subscript means that the two solutions are identical.

$p_r = p_s$. $\bar{\lambda} = \bar{p}/p_r$.

$100p_r$ s $100\bar{p}$	1 0.1 0.15	1 1.0 0.50	1 0.1 0.90	1 3.0 0.90	5 1.0 2.5	5 3.0 3.5	10 0.1 1.5	10 0.3 1.5	10 0.1 9.0	10 1.0 5.0
$\bar{\lambda}$	0.15	0.50	0.90	0.90	0.5	0.7	0.15	0.15	0.9	0.5
N	c	c	c	c	c	c	c	c	c	c
100	Acc.	Acc.	0	0	Acc.	Acc.	Acc.	Acc.	0	Acc.
200	Acc.	Acc.	0	0	Acc.	Acc.	0	Acc.	0	Acc.
300	Acc.	Acc.	0	0	Acc.	Acc.	0	Acc.	1	1
500	Acc.	Acc.	0	1	1	Acc.	1	Acc.	1	2
700	Acc.	Acc.	0	1	1	2	1	Acc.	1	3
1000	Acc.$_0$	Acc.	0	1	2	3	1	Acc.	2	3
2000	0	Acc.	0	3	2_1	6	2	Acc.	2	5
3000	0	1	1	4	4_1	7_1	2	Acc.	3	7
5000	1	2	1	5	6	10_1	3	Acc.	4	9_1
7000	1	3	1	7	8	13	4	Acc.$_3$	5	11_1
10000	1	3	2	9	9_1	16	5	4	6	14
20000	2	5	2	13	14	–	7	6	9	–
30000	2	7	3	17	17	–	9	8	11_1	–
50000	3	9_1	4	–	–	–	11	11	15	–
70000	4	11_1	5	–	–	–	13_1	13_1	18	–

and

$$V\{p\} = \bar{p}\bar{q}/(s+t+1)$$

we get

$$s = \bar{p}(\bar{p}\bar{q} - V\{p\})/V\{p\}. \tag{94}$$

From

$$E\{x^{(\nu)}\} = n^{(\nu)}E\{p^{\nu}\}$$

we obtain

$$E\{x\} = nE\{p\} = n\bar{p}$$

and

$$V\{x\} = n^{(2)}V\{p\} + n\bar{p}\bar{q}.$$

Introducing $E\{x\}$ and $V\{x\}$ in the expressions for $\bar{p}$ and s we find $\bar{p} = E\{x/n\}$ and

$$s = \bar{p}(\bar{p}\bar{q} - V\{x/n\})/(V\{x/n\} - \bar{p}\bar{q}/n). \tag{95}$$

Estimates of p and s are obtained by replacing $E\{x\}$ and $V\{x\}$ by the sample mean and variance.

In the Poisson case $x_1, \ldots, x_k$ represent the number of defects in k samples (or lots) of size m, the distribution of x being $b_\gamma(x,m)$, see (9).

Since $E\{\lambda\} = \bar{\lambda} = s/\tau$ and $V\{\lambda\} = s/\tau^2 = \bar{\lambda}^2/s$ we have $s = \bar{\lambda}^2/V\{\lambda\}$.

From $E\{x^{(\nu)}\} = m^\nu E\{\lambda^\nu\}$ we obtain $E\{x\} = m\bar{\lambda}$ and

$$V\{x\} = m^2 V\{\lambda\} + m\bar{\lambda}.$$

Introducing $E\{x\}$ and $V\{x\}$ in the expressions for $\bar{\lambda}$ and s we find $\bar{\lambda} = E\{x\}/m$ and

$$s = (E\{x\})^2/(V\{x\} - E\{x\}).$$

Estimates of $\bar{\lambda}$ and s are obtained by replacing $E\{x\}$ and $V\{x\}$ by the sample mean and variance.

If we do not have previous inspection results from which we may estimate $(\bar{\lambda},s)$, we have to guess at these parameters from a general (and usually vague) knowledge of the expected quality variation. In this way we may get a reasonable first sampling plan, which in due time may be improved on the basis of inspection results. Guessing at $\bar{\lambda}$ and σ_λ we may determine s from $s = (\bar{\lambda}/\sigma_\lambda)^2$. As a check it is often useful to consider the rejectable part of the prior distribution, see (81), which may be found from the tables in the Appendix.

Example 1.

Published data regarding quality variation are very scarce. In many cases it is, however, rather easy to find the empirical distribution of the fraction defective from inspection records and it turns out that this distribution often is remarkably stable. In the experience of the authors the beta distribution has normally given a reasonably good fit to the observed variation of the fraction defective. Some examples of typical values of the estimated parameters are given below.

Apart from the first case the examples given here all have an average percentage defective larger than 1 and a value of s larger than 1. For items having smaller average

Examples of values of $\bar{p}$ and s estimated from k lots with n observations per lot.

Item	k	n	$100\bar{p}$	s
Wire-wound resistors	421	1500	0.51	0.31
Impedance error in transformers	150	700	1.7	4.9
Insulation tubes	58	30000	5.5	7.7
Control of bottles in glass works	3461	170	1.3	1.0
Breakage during bottling in brewery	73	15000	2.0	5.8
Control of used bottles returned to brewery	205	5000	1.8	7.2
TV tuners	286	100	13	2.2
TV protective glass	179	200	12	2.5
Finish on Radio cabinets	278	60	11	1.3

TABLE 9.

Comparison of exact and approximate binomial sampling plans for $s = 4.9$, $\bar{p} = 0.0165$ and $p_s = p_r = 0.02$.

	Poisson sampling plans			Binominal sampling plans							
	$s = 4.9$, $\bar{\lambda} = 0.83$ and $\lambda_s = 1$.			Approximation from Poisson plan			Exact			Approximation from asymptotic formulas. $n(c) = (c - 0.540)/0.02$, $\sqrt{N(c)} = (n(c) + 245.2)/8.698$. $n = n(c) \pm 0.49/(c + 0.440)$. The first entry is the lot size corresponding to $n = 0$.		
c	$M_{c+0.5}$	m_{cl}	m_{cu}	$N_{c+0.5}$	n_{cl}	n_{cu}	$N_{c+0.5}$	n_{cl}	n_{cu}	$N(c+0.5)$	n	
	11.2	Accept		560	Accept		643	Accept		795	–	
1	15.4	0.561	0.693	770	28	35	817	28	33	1140	8	40
2	23.3	1.44	1.64	1160	72	82	1210	71	80	1560	63	83
3	32.6	2.44	2.61	1630	122	130	1690	121	129	2040	116	130
4	43.4	3.45	3.59	2170	172	180	2240	171	178	2600	167	179
5	55.4	4.45	4.58	2770	222	229	2860	221	227	3220	218	228
6	68.8	5.46	5.57	3440	273	278	3540	271	277	3900	269	277
7	83.4	6.46	6.56	4170	323	328	4290	321	326	4650	320	326
8	99.4	7.47	7.56	4970	374	378	5110	372	376	5470	370	376
9	117	8.47	8.55	5850	424	428	5990	422	426	6350	420	426
10	135	9.47	9.55	6750	474	478	6940	472	475	7300	471	475

percentage defective we shall naturally often find J-shaped prior distributions, i.e. $0 < s \leqq 1$.

We shall discuss the second case in more detail. From the 150 observed values of the fraction defective we compute the average, $\bar{p} = 0.0165$, and the variance, $s_p^2 = 0.0^4 7818$. Replacing the expectations in (95) by these values and inserting $n = 700$ we find $s = 4.9$.

Suppose that $p_r = 0.02$. Since this value is rather small we shall use the Poisson solution as approximation to the optimal sampling plan. We find $\bar{\lambda} = \bar{p}/p_r = 0.83$ and $s = 4.9$. The approximate solution derived from tables of optimum Poisson sampling plans has been shown in Table 9. Furthermore, the optimum binomial plans, exact as well as approximate, see HALD (1968b), have been tabulated.

For a lot size of $N = 700$ the optimum plan is $(n,c) = (30,1)$, the Poisson approximation gives (33,1) and the asymptotic expansion leads to acceptance without inspection.

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Acknowledgements

We are grateful to Professor S. JOHANSEN for discussions which have led to improvement of our original proof of the main theorem.

Our thanks are due to Mr. O. KNUDSEN and Mr. U. MØLLER for help with the programming of the tables and for carrying out numerous calculations necessary for checking the accuracy of the approximation formulas and the interpolation methods.

The computations were carried out at the Northern Europe University Computing Center, Lyngby, Denmark.

We are grateful for a grant from the Carlsberg Foundation covering part of the programming expenses.

APPENDIX

12. Tables of optimal sampling plans for the two-point prior distribution. Description and use of the tables

The tables of sampling plans give c and m as functions of M, where c and m for given M minimize the function

$$R^*(c,m,M) = m + (M-m)\{\gamma_1(1-B(c,m)) + \gamma_2 B(c,rm)\}, \tag{96}$$

$B(c,m) = \sum_{x=0}^{c} e^{-m} m^x/x!$ and (r,γ_1,γ_2) are given constants.

In practice the problem will normally be to minimize the function

$$R(c,m,M) = m + (M-m)\{\gamma_1(1-B(c,m\lambda_1)) + \gamma_2 B(c,m\lambda_2)\}.$$

Since $\lambda_1 R(c,m,M) = R^*(c,m^*,M^*)$ for $m^* = m\lambda_1$, $M^* = M\lambda_1$ and $r = \lambda_2/\lambda_1$ this problem is solved by reading (c,m^*) from the table for the argument M^* and finding the sampling plan as $(c,m^*/\lambda_1)$.

The parameter r equals the ratio of the two mean occurrence rates, λ_1 and λ_2, per inspection unit.

The parameters (γ_1,γ_2) are arbitrary positive numbers. In Sections 3 and 9 it has been shown how (γ_1,γ_2) may be interpreted within the framework of a linear cost model with the three cost functions $k_s(\lambda) = S_1 + S_2\lambda$, $k_a(\lambda) = A_1 + A_2\lambda$ and $k_r(\lambda) = R_1 + R_2\lambda$ and a two-point prior distribution of λ given by $(\lambda_1,\lambda_2,w_1)$. The resulting formulas are summarized below, setting $w_2 = 1 - w_1$.

$$\bar{\lambda} = w_1\lambda_1 + w_2\lambda_2,\ \lambda_r = (R_1 - A_1)/(A_2 - R_2).$$
$$\lambda_0 = w_1\lambda_1 + w_2\lambda_r,\ \lambda_s = \lambda_r + \{S_1 - R_1 + (S_2 - R_2)\bar{\lambda}\}/(A_2 - R_2).$$
$$\gamma_1 = w_1(\lambda_r - \lambda_1)/(\lambda_s - \lambda_0),\ \gamma_2 = w_2(\lambda_2 - \lambda_r)/(\lambda_s - \lambda_0).$$

The tables may also be used for obtaining an approximation to the solution of the analogous binomial problem with regret function

$$R_b(c,n,N) = n + (N-n)\{\gamma_1(1-B(c,n,p_1)) + \gamma_2 B(c,n,p_2)\},$$

where $B(c,n,p) = \sum_{x=0}^{c} \binom{n}{x} p^x q^{n-x}$. By means of the usual approximation, $B(c,n,p) \simeq B(c,np)$, we get $p_1 R_b(c,n,N) \simeq R^*(c,np_1,Np_1)$ with $r = p_2/p_1$. Thus, the binomial problem is solved (approximately) by reading (c,m) from the table for the argument $M = Np_1$ and finding the sampling plan as $(c,m/p_1)$.

For the linear cost model the parameters (γ_1,γ_2) are obtained from the formulas above replacing (λ_1,λ_2) by (p_1,p_2).

The approximation is good for $p_r < 0.05$ and $r > 3$.

The tables contain solutions for $\gamma_1 = 1$ only. For $\gamma_1 < 1$ a good approximation to the solution is obtained by entering the tables with $M^* = M\gamma_1$ and $\gamma_2^* = \gamma_2/\gamma_1$ as arguments.

Tables are given for the following values of γ_2 and r:

γ_2 = 0.04, 0.065, 0.10, 0.15, 0.25, 0.40, 0.65, 1.00.
r = 2.00, 2.25, 2.50, 2.75, 3.0, 3.5, 4.0, 5.0, 6.5, 10, 15, 25.

The solution has been tabulated with the double limitation that $M < 10{,}000$ and $c \leqq 19$.

Each line of the table contains c, $M_{c+0.5}$, m_{cl} *and* m_{cu}. *For* $M_{c-0.5} < M \leqq M_{c+0.5}$ *the optimum acceptance number is c, and the optimum values of m corresponding to* $M_{c-0.5}$ *and* $M_{c+0.5}$ *are* m_{cl} *and* m_{cu}, *respectively. Intermediate values of m may be obtained by linear interpolation with respect to M.*

If the first entry in the column headed sample size is "Accept" then the minimum of R^* is obtained for $m = 0$, i.e. by acceptance without inspection, and $\min R^* = M\gamma_2$ for $M < M^*$, where M^* represents the first entry in the M-column.

Example 2.

Suppose that batches of cloth are inspected for weave defects, each batch consisting of $M = 4000$ yards. Previous inspection results have shown that the mean occurrence rate of defects in a batch may be described by a random variable which takes on the value $\lambda_1 = 0.2$ defects/yard with probability $w_1 = 20/21$, and the value $\lambda_2 = 2.0$ defects/yard with probability $w_2 = 1/21$. The costs of inspecting, accepting or rejecting a yard of quality λ are $k_s(\lambda) = 1.0 + 0.2\lambda$, $k_a(\lambda) = 1.2\lambda$ and $k_r(\lambda) = 0.8 + 0.2\lambda$, respectively.

From the formulas above we obtain $\bar{\lambda} = 0.2857$, $\lambda_r = 0.8$, $\lambda_0 = 0.2286$, $\lambda_s = 1.0$, $\gamma_1 = 0.7407$, $\gamma_2 = 0.0741$ and $r = \lambda_2/\lambda_1 = 10$. Since $\lambda_1 \neq 1$ and $\gamma_1 \neq 1$ we have to use the modified lot size $M^* = M\lambda_1\gamma_1 = 4000 \times 0.2 \times 0.7407 = 593$ and also $\gamma_2^* = \gamma_2/\gamma_1 = 0.1$. The table shows that for $526 < M^* \leqq 1030$ the optimum acceptance number is $c = 8$ and that $1.89 < m^* \leqq 1.95$. Linear interpolation gives that the sample size corresponding to $M^* = 593$ is $m^* = 1.90$. Finally, the sample size in the original units becomes $m = m^*/\lambda_1 = 9.50$ yards.

Interpolation in the tables.

We note the following interpolation rules.

Interpolation with respect to γ_2. Use linear interpolation on $\log M_{c+0.5}$, m_{cl}, and m_{cu} with $\log \gamma_2$ as argument.

Interpolation with respect to r. Use quadratic interpolation on $\log M_{c+0.5}$ in the neighbourhood of the minimum and linear interpolation on $\log M_{c+0.5}$ otherwise. Use linear interpolation on m_{cl} and m_{cu} with r as argument for $r < 5$; for $r > 5$ use linear interpolation with d_0^{-1} as argument.

Example 3. Interpolation with respect to γ_2.

Suppose that the plans for $\gamma_2 = 0.15$ had not been tabulated and that we want to find the optimum plan for $M = 50$ and $r = 5$ by interpolation from the tabulated plans for $\gamma_2 = 0.10$ and 0.25. From the tables we read the following results:

c	$M_{c+0.5}$ $\gamma_2 = 0.10$	$M_{c+0.5}$ $\gamma_2 = 0.25$	Interpolate $\gamma_2 = 0.15$	Exact $\gamma_2 = 0.15$
5	56.2	36.9	46.6	46.2
6	81.4	54.2	68.0	67.4

It will be seen that $M = 50$ leads to $c = 5$ for $\gamma_2 = 0.10$ and $c = 6$ for $\gamma_2 = 0.25$. Linear interpolation on $\log M$ with $\log \gamma_2$ as argument gives a rather good approximation to the exact values. Naturally this result could be improved by using quadratic interpolation. The optimum value of c is found to be 6.

From the tables we find for $c = 6$

γ_2	m_{cl}	m_{cu}	
0.10	2.02	2.08	
0.25	2.24	2.31	
0.15	2.12	2.18	Interpolation
0.15	2.11	2.18	Exact

Finally, linear interpolation to $M = 50$ between (46.6, 2.12) and (68.0, 2.18) gives $m = 2.13$ so that the result is $(m,c) = (2.13, 6)$ as compared to the exact solution (2.12, 6).

Example 4. Interpolation with respect to r.

Suppose that the plans for $r = 3.5$ had not been tabulated and that we want to find the optimum plan for $M = 30$ and $\gamma_2 = 0.25$ by interpolation from the tabulated plans for $r = 2.5$, 3.0 and 4.0. Using Aitken's method for quadratic interpolation we find for $c = 5$

r	$M_{c+0.5}$	$3.5 - r$	$\log M$			Antilog
2.5	30.4	1.0	1.4829	1.4419		
3.0	29.0	0.5	1.4624		1.4671	29.3
4.0	31.4	– 0.5	1.4969	1.4797		

The result is $M = 29.3$ as compared to the exact solution $M = 29.7$. By the same method we find for $c = 6$ that $M = 38.5$ as compared to the exact $M = 38.9$. (Since the variation of M in this example is rather small it does not matter much whether we use $\log M$ or M itself for the interpolation).

The result shows that the optimum acceptance number for $M = 30$ is $c = 6$.

By linear interpolation with respect to r we get from the tables for $c = 6$

r	m_{cl}	m_{cu}	
3.0	2.86	2.92	
4.0	2.52	2.58	
3.5	2.69	2.75	Interpolation
3.5	2.68	2.75	Exact

Finally, linear interpolation to $M = 30$ between (29.3, 2.69), and (38.5, 2.75) leads to the plan (2.69, 6) as compared to the exact solution (2.68, 6).

Suppose now that the problem is to find m_{cu} for $c = 6$ corresponding to $r = 15$ from the values for $r = 10$ and 25, viz. 1.54 and 0.84. Linear interpolation gives 1.31, which differs rather much from the correct result 1.19. Noting that d_0^{-1} for $r = 10$, 15, 25 equals 0.2558, 0.1934, 0.1341, we obtain by linear interpolation $m_{cu} = 1.18$.

Use of the asymptotic expansions.

To make the asymptotic expansions given in Section 9 easier to use the most important coefficients have been tabulated. We shall summarize the procedure below.

Construction of a table of optimum sampling plans. For successive values of c, $c = 0,1,2,\ldots$, compute $m(c)$ and $m(c+0.5)$ from

$$m(c) = \left\{c - d_2^* - (\ln r)^{-1}\left(\ln\frac{\gamma_1}{\gamma_2}\right)\right\}/d_0. \tag{97}$$

Compute $M(c+0.5)$ from

$$\ln(M-m) = e_0 m + \tfrac{1}{2}\ln m + h + \left(\frac{\ln d_0}{\ln r}\right)\left(\ln\frac{\gamma_1}{\gamma_2}\right) - \ln\gamma_1, \tag{98}$$

with $m = m(c+0.5)$. Finally, m_{cl} is determined from

$$m_{cl} \simeq m(c) - \frac{1}{2rd_0}\left(e_0 + \frac{1}{2m(c)}\right), \tag{99}$$

and m_{cu} is obtained by changing the sign of the correction term above.

Construction of isolated sampling plans. Determine $m(M)$ from

$$e_0 m = f(M) - g(M)\left\{h - 0.5\ln e_0 + \left(\frac{\ln d_0}{\ln r}\right)\left(\ln\frac{\gamma_1}{\gamma_2}\right) - \ln\gamma_1\right\}, \tag{100}$$

and then

$$c(M) = m(M)d_0 + d_2^* + (\ln r)^{-1}\left(\ln\frac{\gamma_1}{\gamma_2}\right).$$

Round $c(M)$ to the nearest integer, c say, and determine $m(c)$ from (97) and $M(c)$ from (98) with $m = m(c)$. Finally, the sample size is found from

$$m \simeq m(c) + \{M - M(c)\}/\{rM(c)\}.$$

Example 5.

Suppose that we want to determine the optimum sampling for $M = 100$, $\gamma_1 = 1$, $\gamma_2 = 0.65$ and $r = 5.5$ without using the tables of optimum sampling plans. From

Table of constants in the asymptotic relation between c and m.

$$c = md_0 + d_2^* + (\ln r)^{-1}\left(\ln\frac{\gamma_1}{\gamma_2}\right).$$

r	d_0	d_2^*	$(\ln r)^{-1}$	d_0^{-1}
1.5	1.2332	– 0.5006	2.46630	0.81093
2.0	1.4427	– 0.5017	1.44270	0.69315
2.5	1.6370	– 0.5029	1.09136	0.61086
3.0	1.8205	– 0.5041	0.91024	0.54931
3.5	1.9956	– 0.5053	0.79824	0.50111
4.0	2.1640	– 0.5065	0.72135	0.46210
4.5	2.3270	– 0.5076	0.66486	0.42974
5.0	2.4853	– 0.5087	0.62133	0.40236
5.5	2.6397	– 0.5097	0.58660	0.37883
6.0	2.7906	– 0.5107	0.55811	0.35835
6.5	2.9383	– 0.5117	0.53424	0.34033
7.0	3.0834	– 0.5126	0.51390	0.32432
7.5	3.2260	– 0.5134	0.49630	0.30999
8.0	3.3663	– 0.5142	0.48090	0.29706
8.5	3.5046	– 0.5150	0.46728	0.28534
9.0	3.6410	– 0.5158	0.45512	0.27465
9.5	3.7756	– 0.5166	0.44419	0.26486
10.0	3.9087	– 0.5173	0.43429	0.25584
11.0	4.1703	– 0.5186	0.41703	0.23979
12.0	4.4267	– 0.5199	0.40243	0.22590
13.0	4.6785	– 0.5211	0.38987	0.21375
14.0	4.9260	– 0.5223	0.37892	0.20300
15.0	5.1698	– 0.5233	0.36927	0.19343
16.0	5.4101	– 0.5244	0.36067	0.18484
17.0	5.6473	– 0.5253	0.35296	0.17708
18.0	5.8816	– 0.5263	0.34598	0.17002
19.0	6.1132	– 0.5272	0.33962	0.16358
20.0	6.3424	– 0.5280	0.33381	0.15767
21.0	6.5692	– 0.5289	0.32846	0.15223
22.0	6.7938	– 0.5297	0.32352	0.14719
23.0	7.0164	– 0.5304	0.31893	0.14252
24.0	7.2371	– 0.5312	0.31466	0.13818
25.0	7.4560	– 0.5319	0.31067	0.13412

(100), $\ln(\gamma_1/\gamma_2) = 0.43078$ and the auxiliary tables we find $m(M) = \{3.9245 - 0.89143(0.6824 + 0.56939 \times 0.43078)\}/0.92255 = 3.358$, such that $c(M) = 3.358 \times 2.640 - 0.5097 + 0.5866 \times 0.4308 = 8.61$, which gives $c = 9$. Now $m(9)$ is determined as $m(9) = (9 + 0.5097 - 0.5866 \times 0.4308)/2.6397 = 3.507$, and $M(9)$ is found from $\ln(M - 3.507) = 0.9226 \times 3.507 + 0.6274 + 0.6421 + 0.5694 \times 0.4308 = 4.750$, such that we have $M(9) = 115.6$ and finally $m = 3.507 + (100 - 115.6)/(5.5 \times 115.6) = 3.48$.

Table of constants in the asymptotic relation between M and m.

$$\ln(M - m) = e_0 m + \tfrac{1}{2}\ln m + h + \left(\frac{\ln d_0}{\ln r}\right)\left(\ln\frac{\gamma_1}{\gamma_2}\right) - \ln\gamma_1.$$

$$e_0 m = f(M) - g(M)\left\{h - 0.5\ln e_0 + \left(\frac{\ln d_0}{\ln r}\right)\left(\ln\frac{\gamma_1}{\gamma_2}\right) - \ln\gamma_1\right\}.$$

r	e_0	h	$(\ln d_0)/(\ln r)$	$h - 0.5(\ln e_0)$
1.5	0.025284	2.4129	0.51687	4.2517
2.0	0.086071	1.8036	0.52877	3.0299
2.5	0.16984	1.4675	0.53792	2.3539
3.0	0.27017	1.2391	0.54532	1.8935
3.5	0.38324	1.0680	0.55153	1.5475
4.0	0.50655	0.9321	0.55686	1.2721
4.5	0.63834	0.8198	0.56153	1.0443
5.0	0.77734	0.7246	0.56567	0.8505
5.5	0.92255	0.6421	0.56939	0.6824
6.0	1.0732	0.5694	0.57276	0.5341
6.5	1.2287	0.5046	0.57583	0.4016
7.0	1.3886	0.4462	0.57866	0.2820
7.5	1.5524	0.3931	0.58128	0.1732
8.0	1.7197	0.3444	0.58372	0.0733
8.5	1.8904	0.2995	0.58599	– 0.0189
9.0	2.0641	0.2579	0.58813	– 0.1044
9.5	2.2405	0.2192	0.59013	– 0.1842
10.0	2.4196	0.1829	0.59203	– 0.2589
11.0	2.7849	0.1168	0.59552	– 0.3953
12.0	3.1587	0.0578	0.59868	– 0.5173
13.0	3.5403	0.0045	0.60156	– 0.6276
14.0	3.9286	– 0.0441	0.60420	– 0.7282
15.0	4.3233	– 0.0886	0.60665	– 0.8206
16.0	4.7236	– 0.1297	0.60891	– 0.9060
17.0	5.1292	– 0.1678	0.61103	– 0.9853
18.0	5.5396	– 0.2034	0.61301	– 1.0594
19.0	5.9545	– 0.2368	0.61487	– 1.1288
20.0	6.3736	– 0.2681	0.61663	– 1.1942
21.0	6.7966	– 0.2976	0.61829	– 1.2558
22.0	7.2232	– 0.3256	0.61986	– 1.3143
23.0	7.6534	– 0.3521	0.62136	– 1.3697
24.0	8.0868	– 0.3774	0.62278	– 1.4225
25.0	8.5233	– 0.4014	0.62414	– 1.4728

Table of $f(M)$ and $g(M)$.

$f(M) = \ln M - \frac{1}{2}(\ln\ln M)\,g(M)$. $g(M) = 1 - (2\ln M)^{-1}$.

M	$f(M)$	$g(M)$	M	$f(M)$	$g(M)$
2.0	0.7442	0.27865	7.0	1.6986	0.74305
2.1	0.7906	0.32609	7.1	1.7094	0.74491
2.2	0.8319	0.36585	7.2	1.7202	0.74672
2.3	0.8694	0.39969	7.3	1.7307	0.74848
2.4	0.9040	0.42888	7.4	1.7412	0.75018
2.5	0.9361	0.45432	7.5	1.7515	0.75185
2.6	0.9664	0.47672	7.6	1.7618	0.75347
2.7	0.9949	0.49660	7.7	1.7718	0.75505
2.8	1.0221	0.51438	7.8	1.7818	0.75659
2.9	1.0481	0.53039	7.9	1.7917	0.75809
3.0	1.0730	0.54488	8.0	1.8014	0.75955
3.1	1.0970	0.55807	8.1	1.8110	0.76098
3.2	1.1201	0.57013	8.2	1.8206	0.76237
3.3	1.1424	0.58121	8.3	1.8300	0.76373
3.4	1.1641	0.59143	8.4	1.8393	0.76506
3.5	1.1851	0.60088	8.5	1.8485	0.76636
3.6	1.2055	0.60966	8.6	1.8576	0.76763
3.7	1.2253	0.61783	8.7	1.8667	0.76887
3.8	1.2446	0.62547	8.8	1.8756	0.77009
3.9	1.2635	0.63262	8.9	1.8844	0.77128
4.0	1.2819	0.63933	9.0	1.8932	0.77244
4.1	1.2998	0.64564	9.1	1.9019	0.77358
4.2	1.3174	0.65159	9.2	1.9104	0.77469
4.3	1.3346	0.65721	9.3	1.9189	0.77579
4.4	1.3514	0.66253	9.4	1.9273	0.77686
4.5	1.3678	0.66757	9.5	1.9357	0.77791
4.6	1.3840	0.67236	9.6	1.9439	0.77893
4.7	1.3998	0.67691	9.7	1.9521	0.77994
4.8	1.4153	0.68125	9.8	1.9602	0.78093
4.9	1.4305	0.68538	9.9	1.9682	0.78190
5.0	1.4454	0.68933	10.0	1.9761	0.78285
5.1	1.4601	0.69311	10.2	1.9918	0.78470
5.2	1.4745	0.69672	10.4	2.0072	0.78649
5.3	1.4887	0.70019	10.6	2.0223	0.78821
5.4	1.5026	0.70351	10.8	2.0372	0.78988
5.5	1.5163	0.70670	11.0	2.0518	0.79148
5.6	1.5297	0.70977	11.2	2.0662	0.79304
5.7	1.5430	0.71272	11.4	2.0803	0.79454
5.8	1.5560	0.71556	11.6	2.0942	0.79600
5.9	1.5689	0.71830	11.8	2.1079	0.79741
6.0	1.5815	0.72094	12.0	2.1214	0.79879
6.1	1.5940	0.72350	12.2	2.1346	0.80011
6.2	1.6063	0.72596	12.4	2.1477	0.80141
6.3	1.6184	0.72834	12.6	2.1606	0.80266
6.4	1.6303	0.73065	12.8	2.1733	0.80388
6.5	1.6421	0.73288	13.0	2.1858	0.80506
6.6	1.6537	0.73504	13.2	2.1981	0.80622
6.7	1.6651	0.73713	13.4	2.2103	0.80734
6.8	1.6764	0.73917	13.6	2.2223	0.80843
6.9	1.6876	0.74114	13.8	2.2341	0.80950

Table of $f(M)$ and $g(M)$.

$f(M) = \ln M - \frac{1}{2}(\ln \ln M)\, g(M)$. $g(M) = 1 - (2 \ln M)^{-1}$.

M	$f(M)$	$g(M)$	M	$f(M)$	$g(M)$
14.0	2.2458	0.81054	40.0	3.1247	0.86446
14.2	2.2573	0.81155	41.0	3.1459	0.86536
14.4	2.2687	0.81254	42.0	3.1666	0.86623
14.6	2.2799	0.81350	43.0	3.1869	0.86706
14.8	2.2910	0.81445	44.0	3.2067	0.86787
15.0	2.3019	0.81537	45.0	3.2261	0.86865
15.2	2.3127	0.81626	46.0	3.2450	0.86941
15.4	2.3234	0.81714	47.0	3.2636	0.87013
15.6	2.3339	0.81800	48.0	3.2818	0.87084
15.8	2.3444	0.81884	49.0	3.2997	0.87153
16.0	2.3547	0.81966	50.0	3.3172	0.87219
16.2	2.3648	0.82047	51.0	3.3343	0.87283
16.4	2.3749	0.82126	52.0	3.3512	0.87346
16.6	2.3848	0.82203	53.0	3.3677	0.87406
16.8	2.3947	0.82278	54.0	3.3839	0.87465
17.0	2.4044	0.82352	55.0	3.3999	0.87523
17.2	2.4140	0.82425	56.0	3.4155	0.87579
17.4	2.4235	0.82496	57.0	3.4309	0.87633
17.6	2.4330	0.82566	58.0	3.4461	0.87686
17.8	2.4423	0.82634	59.0	3.4610	0.87738
18.0	2.4515	0.82701	60.0	3.4756	0.87788
18.2	2.4606	0.82767	61.0	3.4900	0.87837
18.4	2.4696	0.82832	62.0	3.5042	0.87885
18.6	2.4786	0.82895	63.0	3.5182	0.87932
18.8	2.4874	0.82958	64.0	3.5319	0.87978
19.0	2.4962	0.83019	65.0	3.5455	0.88022
19.2	2.5048	0.83079	66.0	3.5588	0.88066
19.4	2.5134	0.83138	67.0	3.5720	0.88109
19.6	2.5219	0.83196	68.0	3.5849	0.88150
19.8	2.5304	0.83253	69.0	3.5977	0.88191
20.0	2.5387	0.83310	70.0	3.6103	0.88231
21.0	2.5793	0.83577	71.0	3.6228	0.88270
22.0	2.6181	0.83824	72.0	3.6350	0.88309
23.0	2.6552	0.84054	73.0	3.6471	0.88346
24.0	2.6909	0.84267	74.0	3.6591	0.88383
25.0	2.7252	0.84467	75.0	3.6708	0.88419
26.0	2.7582	0.84654	76.0	3.6825	0.88455
27.0	2.7900	0.84829	77.0	3.6940	0.88489
28.0	2.8207	0.84995	78.0	3.7053	0.88523
29.0	2.8504	0.85151	79.0	3.7165	0.88557
30.0	2.8791	0.85299	80.0	3.7276	0.88590
31.0	2.9069	0.85440	81.0	3.7385	0.88622
32.0	2.9339	0.85573	82.0	3.7493	0.88654
33.0	2.9601	0.85700	83.0	3.7600	0.88685
34.0	2.9856	0.85821	84.0	3.7705	0.88715
35.0	3.0103	0.85937	85.0	3.7809	0.88745
36.0	3.0344	0.86047	86.0	3.7913	0.88775
37.0	3.0578	0.86153	87.0	3.8014	0.88804
38.0	3.0807	0.86255	88.0	3.8115	0.88833
39.0	3.1029	0.86352	89.0	3.8215	0.88861

Table of $f(M)$ and $g(M)$.

$f(M) = \ln M - \frac{1}{2}(\ln\ln M)\, g(M)$. $g(M) = 1 - (2\ln M)^{-1}$.

M	$f(M)$	$g(M)$	M	$f(M)$	$g(M)$
90.0	3.8314	0.88888	500	5.3746	0.91954
91.0	3.8411	0.88916	510	5.3928	0.91980
92.0	3.8508	0.88942	520	5.4105	0.92005
93.0	3.8603	0.88969	530	5.4279	0.92029
94.0	3.8698	0.88995	540	5.4450	0.92053
95.0	3.8791	0.89020	550	5.4618	0.92076
96.0	3.8884	0.89046	560	5.4783	0.92099
97.0	3.8975	0.89070	570	5.4945	0.92121
98.0	3.9066	0.89095	580	5.5105	0.92142
99.0	3.9156	0.89119	590	5.5261	0.92163
100	3.9245	0.89143	600	5.5415	0.92184
110	4.0090	0.89363	610	5.5567	0.92204
120	4.0863	0.89556	620	5.5716	0.92224
130	4.1575	0.89728	630	5.5863	0.92243
140	4.2236	0.89882	640	5.6007	0.92262
150	4.2853	0.90021	650	5.6150	0.92280
160	4.3430	0.90148	660	5.6290	0.92298
170	4.3973	0.90264	670	5.6428	0.92316
180	4.4486	0.90372	680	5.6564	0.92334
190	4.4972	0.90471	690	5.6698	0.92351
200	4.5433	0.90563	700	5.6830	0.92368
210	4.5872	0.90649	710	5.6960	0.92384
220	4.6291	0.90730	720	5.7089	0.92400
230	4.6692	0.90806	730	5.7216	0.92416
240	4.7076	0.90877	740	5.7341	0.92432
250	4.7445	0.90944	750	5.7464	0.92447
260	4.7800	0.91008	760	5.7586	0.92462
270	4.8141	0.91069	770	5.7706	0.92477
280	4.8470	0.91127	780	5.7825	0.92492
290	4.8788	0.91181	790	5.7942	0.92506
300	4.9095	0.91234	800	5.8058	0.92520
310	4.9393	0.91284	810	5.8172	0.92534
320	4.9681	0.91332	820	5.8285	0.92548
330	4.9960	0.91378	830	5.8396	0.92561
340	5.0231	0.91422	840	5.8507	0.92574
350	5.0495	0.91465	850	5.8616	0.92587
360	5.0751	0.91505	860	5.8723	0.92600
370	5.1000	0.91545	870	5.8830	0.92613
380	5.1243	0.91583	880	5.8935	0.92625
390	5.1479	0.91619	890	5.9039	0.92638
400	5.1710	0.91655	900	5.9142	0.92650
410	5.1935	0.91689	910	5.9244	0.92662
420	5.2155	0.91722	920	5.9345	0.92673
430	5.2369	0.91754	930	5.9444	0.92685
440	5.2579	0.91785	940	5.9543	0.92696
450	5.2784	0.91816	950	5.9641	0.92708
460	5.2985	0.91845	960	5.9737	0.92719
470	5.3181	0.91874	970	5.9833	0.92730
480	5.3373	0.91901	980	5.9927	0.92741
490	5.3562	0.91928	990	6.0021	0.92751

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.040$.

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							266	ACCEPT	
11							270	4.91	4.92
12							297	5.49	5.53
13				366	ACCEPT		329	6.10	6.14
14				367	6.86	6.86	366	6.71	6.75
15				396	7.49	7.51	408	7.32	7.36
16				428	8.13	8.16	455	7.93	7.97
17				464	8.78	8.81	508	8.54	8.58
18				505	9.43	9.46	568	9.15	9.19
19	543	ACCEPT		549	10.1	10.1	636	9.76	9.80

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.040$.

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							112	ACCEPT	
6							121	2.00	2.02
7				163	ACCEPT		146	2.47	2.52
8	204	ACCEPT		177	3.08	3.10	178	2.97	3.02
9	209	3.69	3.69	204	3.61	3.65	218	3.46	3.51
10	235	4.23	4.27	237	4.16	4.20	268	3.96	4.01
11	266	4.81	4.85	277	4.70	4.75	331	4.46	4.51
12	303	5.38	5.42	325	5.25	5.29	408	4.96	5.01
13	346	5.96	6.00	382	5.80	5.84	504	5.46	5.51
14	396	6.54	6.58	449	6.35	6.39	623	5.96	6.01
15	453	7.11	7.15	528	6.89	6.94	768	6.46	6.51
16	520	7.69	7.73	621	7.44	7.49	948	6.96	7.01
17	597	8.27	8.31	730	7.99	8.04	1170	7.47	7.52
18	685	8.85	8.89	859	8.54	8.58	1440	7.97	8.02
19	786	9.42	9.46	1010	9.09	9.13	1780	8.47	8.52

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.040$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							31.7	ACCEPT	
3				52.3	ACCEPT		45.1	0.605	0.660
4	82.8	ACCEPT		67.8	1.01	1.06	70.3	0.927	0.996
5	93.8	1.49	1.52	93.9	1.39	1.46	111	1.26	1.33
6	118	1.92	1.98	132	1.79	1.86	176	1.60	1.67
7	152	2.38	2.44	185	2.19	2.26	279	1.94	2.01
8	195	2.84	2.90	262	2.59	2.66	441	2.28	2.35
9	252	3.30	3.36	369	3.00	3.06	695	2.62	2.69
10	326	3.76	3.82	519	3.40	3.46	1090	2.96	3.03
11	421	4.22	4.28	731	3.80	3.86	1720	3.30	3.37
12	544	4.69	4.74	1030	4.20	4.26	2700	3.64	3.71
13	703	5.15	5.20	1440	4.60	4.66	4220	3.98	4.05
14	909	5.61	5.66	2030	5.01	5.07	6600	4.32	4.39
15	1170	6.07	6.13	2840	5.41	5.47	10300	4.66	4.73
16	1510	6.53	6.59	3970	5.81	5.87			
17	1950	6.99	7.05	5560	6.21	6.27			
18	2520	7.46	7.51	7770	6.61	6.67			
19	3240	7.92	7.97	10900	7.02	7.08			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.040$.

	r =10.00			r =15.00			r =25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				7.40	ACCEPT		3.67	ACCEPT	
1	15.0	ACCEPT		10.8	0.0868	0.110	9.58	0.0620	0.103
2	25.3	0.278	0.334	26.5	0.229	0.295	34.0	0.176	0.232
3	49.6	0.513	0.584	66.8	0.419	0.485	119	0.310	0.364
4	98.1	0.766	0.836	167	0.612	0.676	409	0.445	0.497
5	194	1.02	1.09	414	0.806	0.868	1390	0.579	0.630
6	380	1.28	1.34	1020	0.999	1.06	4670	0.713	0.764
7	744	1.53	1.60	2490	1.19	1.25	15600	0.847	0.898
8	1450	1.79	1.85	6050	1.39	1.45			
9	2810	2.04	2.11	14600	1.58	1.64			
10	5440	2.30	2.36						
11	10500	2.56	2.62						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.065$.

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							142	ACCEPT	
9							149	4.01	4.02
10							166	4.60	4.63
11				196	ACCEPT		185	5.20	5.24
12				211	5.93	5.96	208	5.81	5.85
13				229	6.58	6.60	233	6.42	6.46
14				251	7.22	7.25	262	7.03	7.07
15	290	ACCEPT		274	7.87	7.90	295	7.64	7.68
16	302	8.72	8.73	300	8.52	8.55	332	8.25	8.29
17	321	9.40	9.43	329	9.16	9.20	374	8.86	8.90
18	342	10.1	10.1	361	9.81	9.84	421	9.47	9.51
19	365	10.8	10.8	396	10.5	10.5	474	10.1	10.1

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.065$.

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							59.4	ACCEPT	
5				86.6	ACCEPT		68.7	1.68	1.71
6	109	ACCEPT		89.9	2.24	2.25	84.6	2.16	2.21
7	110	2.81	2.82	105	2.75	2.80	105	2.66	2.71
8	125	3.35	3.39	123	3.30	3.35	131	3.15	3.21
9	143	3.93	3.97	146	3.85	3.89	163	3.65	3.71
10	165	4.50	4.55	172	4.39	4.44	203	4.15	4.21
11	190	5.08	5.12	204	4.94	4.99	252	4.65	4.71
12	220	5.66	5.70	242	5.49	5.54	314	5.15	5.21
13	254	6.23	6.28	286	6.04	6.08	389	5.66	5.71
14	293	6.81	6.85	339	6.59	6.63	483	6.16	6.21
15	338	7.39	7.43	401	7.14	7.18	599	6.66	6.71
16	390	7.97	8.01	474	7.68	7.73	742	7.16	7.21
17	450	8.54	8.59	560	8.23	8.28	918	7.66	7.71
18	519	9.12	9.16	662	8.78	8.83	1140	8.16	8.21
19	599	9.70	9.74	782	9.33	9.38	1400	8.66	8.71

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.065$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							16.3	ACCEPT	
2				27.4	ACCEPT		20.9	0.379	0.415
3	43.9	ACCEPT		34.8	0.738	0.783	33.5	0.675	0.747
4	51.5	1.19	1.22	49.4	1.11	1.18	54.2	1.01	1.08
5	66.4	1.62	1.68	70.6	1.51	1.58	87.3	1.35	1.42
6	86.6	2.08	2.14	101	1.91	1.98	140	1.69	1.76
7	113	2.54	2.60	144	2.31	2.38	224	2.03	2.10
8	148	3.00	3.06	205	2.71	2.78	356	2.37	2.44
9	193	3.46	3.52	292	3.12	3.18	564	2.71	2.77
10	252	3.92	3.98	413	3.52	3.58	891	3.05	3.11
11	328	4.38	4.44	584	3.92	3.98	1400	3.39	3.45
12	426	4.85	4.90	825	4.32	4.38	2210	3.73	3.79
13	553	5.31	5.36	1160	4.72	4.79	3470	4.07	4.13
14	717	5.77	5.82	1630	5.13	5.19	5430	4.41	4.47
15	929	6.23	6.29	2300	5.53	5.59	8490	4.75	4.81
16	1200	6.69	6.75	3220	5.93	5.99	13300	5.09	5.15
17	1550	7.16	7.21	4510	6.33	6.39			
18	2010	7.62	7.67	6320	6.74	6.79			
19	2590	8.08	8.13	8840	7.14	7.20			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.065$.

	r = 10.00			r = 15.00			r = 25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							1.50	ACCEPT	
0	7.46	ACCEPT		3.98	ACCEPT		2.17	0.0000	0.0052
1	9.63	0.119	0.142	8.13	0.0955	0.143	7.77	0.0651	0.122
2	19.4	0.312	0.386	21.4	0.260	0.329	28.6	0.195	0.252
3	39.5	0.563	0.636	55.2	0.453	0.519	101	0.330	0.384
4	79.6	0.818	0.889	140	0.646	0.710	351	0.465	0.517
5	159	1.07	1.14	348	0.840	0.902	1200	0.599	0.650
6	314	1.33	1.40	861	1.03	1.09	4030	0.733	0.784
7	617	1.59	1.65	2110	1.23	1.29	13500	0.867	0.918
8	1210	1.84	1.91	5140	1.42	1.48			
9	2350	2.10	2.16	12500	1.61	1.67			
10	4560	2.35	2.42						
11	8800	2.61	2.67						
12	17000	2.86	2.93						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.10.$

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							78.6	ACCEPT	
7							83.4	3.07	3.09
8				108	ACCEPT		94.1	3.66	3.70
9				113	4.34	4.36	107	4.27	4.31
10				125	4.97	5.01	121	4.88	4.92
11	161	ACCEPT		137	5.62	5.65	137	5.49	5.53
12	163	6.39	6.40	151	6.27	6.30	156	6.10	6.14
13	174	7.06	7.09	167	6.91	6.95	177	6.70	6.74
14	186	7.75	7.78	185	7.56	7.60	200	7.31	7.35
15	200	8.44	8.47	204	8.21	8.24	227	7.93	7.96
16	215	9.14	9.16	225	8.86	8.89	257	8.54	8.57
17	231	9.83	9.86	249	9.51	9.54	291	9.15	9.18
18	248	10.5	10.5	274	10.2	10.2	328	9.76	9.79
19	267	11.2	11.2	302	10.8	10.8	371	10.4	10.4

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.10.$

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							32.8	ACCEPT	
4				47.8	ACCEPT		40.0	1.34	1.39
5	60.2	ACCEPT		54.0	1.89	1.92	50.5	1.83	1.89
6	67.6	2.45	2.49	64.5	2.42	2.47	64.0	2.32	2.38
7	78.5	3.02	3.06	77.3	2.96	3.01	80.9	2.82	2.88
8	91.5	3.59	3.64	92.7	3.51	3.56	102	3.32	3.38
9	107	4.17	4.22	111	4.06	4.11	128	3.82	3.88
10	125	4.75	4.79	133	4.60	4.65	161	4.32	4.38
11	145	5.32	5.37	159	5.15	5.20	201	4.82	4.88
12	169	5.90	5.94	189	5.70	5.75	252	5.33	5.38
13	197	6.48	6.52	226	6.25	6.30	314	5.83	5.88
14	228	7.06	7.10	268	6.80	6.85	390	6.33	6.38
15	265	7.63	7.68	319	7.35	7.40	485	6.83	6.88
16	307	8.21	8.25	378	7.90	7.94	603	7.33	7.38
17	356	8.79	8.83	449	8.45	8.49	747	7.83	7.88
18	412	9.37	9.41	531	9.00	9.04	926	8.33	8.38
19	476	9.94	9.99	629	9.55	9.59	1150	8.83	8.88

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.10$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							8.95	ACCEPT	
1				14.8	ACCEPT		9.61	0.158	0.167
2	24.0	ACCEPT		17.9	0.463	0.497	15.8	0.415	0.492
3	28.6	0.871	0.911	26.2	0.816	0.889	26.5	0.746	0.824
4	38.0	1.30	1.37	38.5	1.21	1.29	43.9	1.09	1.16
5	50.7	1.76	1.82	56.2	1.61	1.68	71.6	1.43	1.50
6	67.5	2.22	2.28	81.4	2.02	2.08	116	1.77	1.84
7	89.4	2.68	2.74	117	2.42	2.48	186	2.11	2.17
8	118	3.14	3.20	168	2.82	2.88	297	2.45	2.51
9	155	3.60	3.66	240	3.22	3.29	472	2.79	2.85
10	203	4.06	4.12	341	3.62	3.69	748	3.13	3.19
11	266	4.53	4.58	483	4.03	4.09	1180	3.47	3.53
12	347	4.99	5.04	683	4.43	4.49	1860	3.81	3.87
13	452	5.45	5.51	965	4.83	4.89	2920	4.15	4.21
14	587	5.91	5.97	1360	5.23	5.29	4590	4.49	4.55
15	762	6.37	6.43	1910	5.64	5.70	7180	4.83	4.89
16	988	6.84	6.89	2690	6.04	6.10	11200	5.17	5.23
17	1280	7.30	7.35	3770	6.44	6.50			
18	1660	7.76	7.81	5280	6.84	6.90			
19	2140	8.22	8.28	7400	7.25	7.30			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.10$.

	r =10.00			r =15.00			r =25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				1.80	ACCEPT		0.600	ACCEPT	
0	4.09	ACCEPT		2.40	0.0000	0.0060	1.58	0.0000	0.0186
1	7.29	0.133	0.187	6.60	0.101	0.172	6.61	0.0765	0.138
2	15.7	0.353	0.432	18.1	0.288	0.358	24.8	0.213	0.269
3	32.8	0.609	0.683	47.1	0.483	0.548	88.5	0.348	0.401
4	66.9	0.865	0.936	120	0.677	0.740	308	0.482	0.534
5	134	1.12	1.19	301	0.870	0.932	1050	0.617	0.668
6	267	1.38	1.44	745	1.06	1.12	3560	0.751	0.801
7	526	1.63	1.70	1830	1.26	1.32	11900	0.885	0.935
8	1030	1.89	1.95	4470	1.45	1.51			
9	2010	2.15	2.21	10900	1.64	1.70			
10	3910	2.40	2.46						
11	7560	2.66	2.72						
12	14600	2.91	2.98						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.15.$

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							42.9	ACCEPT	
5							45.6	2.13	2.15
6				59.5	ACCEPT		52.5	2.71	2.75
7				65.6	3.36	3.39	60.7	3.31	3.36
8	88.7	ACCEPT		73.3	4.00	4.04	70.0	3.92	3.97
9	91.3	4.72	4.73	82.1	4.64	4.68	80.7	4.53	4.58
10	98.8	5.39	5.42	91.9	5.29	5.33	92.7	5.14	5.19
11	107	6.08	6.11	103	5.94	5.98	106	5.75	5.79
12	116	6.77	6.80	115	6.59	6.62	122	6.36	6.40
13	126	7.46	7.49	128	7.23	7.27	139	6.97	7.01
14	137	8.15	8.18	142	7.88	7.92	159	7.58	7.62
15	149	8.85	8.88	158	8.53	8.57	180	8.19	8.23
16	161	9.54	9.57	176	9.18	9.21	205	8.80	8.84
17	175	10.2	10.3	195	9.83	9.86	233	9.41	9.45
18	189	10.9	11.0	216	10.5	10.5	264	10.0	10.1
19	205	11.6	11.6	239	11.1	11.2	299	10.6	10.7

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.15.$

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				26.0	ACCEPT		17.8	ACCEPT	
3	32.7	ACCEPT		26.6	1.03	1.04	23.0	0.992	1.06
4	35.2	1.55	1.57	32.5	1.52	1.58	30.1	1.48	1.55
5	41.8	2.09	2.15	40.0	2.06	2.12	39.2	1.98	2.05
6	49.8	2.67	2.72	49.0	2.61	2.67	50.5	2.48	2.54
7	59.3	3.24	3.29	59.9	3.16	3.21	64.7	2.98	3.04
8	70.3	3.82	3.87	72.8	3.71	3.76	82.3	3.48	3.54
9	83.1	4.40	4.45	88.1	4.26	4.31	104	3.98	4.04
10	97.9	4.97	5.02	106	4.80	4.86	131	4.48	4.54
11	115	5.55	5.60	128	5.35	5.40	165	4.98	5.04
12	135	6.13	6.18	153	5.90	5.95	207	5.49	5.54
13	158	6.71	6.75	183	6.45	6.50	258	5.99	6.04
14	184	7.29	7.33	218	7.00	7.05	322	6.49	6.54
15	214	7.86	7.91	260	7.55	7.60	402	6.99	7.04
16	249	8.44	8.48	310	8.10	8.15	499	7.49	7.54
17	289	9.02	9.06	368	8.65	8.69	620	7.99	8.04
18	335	9.60	9.64	436	9.20	9.24	770	8.49	8.54
19	388	10.2	10.2	517	9.75	9.79	954	8.99	9.04

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.15$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				7.89	ACCEPT		4.60	ACCEPT	
1	12.8	ACCEPT		8.70	0.198	0.213	7.02	0.183	0.239
2	15.5	0.554	0.594	13.5	0.514	0.597	12.6	0.475	0.563
3	21.5	0.969	1.04	20.7	0.908	0.989	21.8	0.815	0.896
4	29.7	1.43	1.50	31.2	1.31	1.39	36.6	1.16	1.23
5	40.4	1.89	1.96	46.2	1.71	1.78	60.2	1.50	1.57
6	54.5	2.35	2.42	67.4	2.12	2.18	97.8	1.84	1.91
7	72.9	2.81	2.87	97.6	2.52	2.58	158	2.18	2.25
8	96.8	3.27	3.34	140	2.92	2.98	252	2.52	2.59
9	128	3.74	3.80	201	3.32	3.39	402	2.86	2.93
10	168	4.20	4.26	286	3.72	3.79	637	3.20	3.26
11	220	4.66	4.72	407	4.13	4.19	1010	3.54	3.60
12	288	5.12	5.18	576	4.53	4.59	1590	3.88	3.94
13	376	5.58	5.64	815	4.93	4.99	2500	4.22	4.28
14	490	6.05	6.10	1150	5.33	5.39	3930	4.56	4.62
15	637	6.51	6.56	1620	5.74	5.80	6150	4.90	4.96
16	827	6.97	7.02	2280	6.14	6.20	9620	5.24	5.31
17	1070	7.43	7.49	3200	6.54	6.60	15000	5.58	5.65
18	1390	7.89	7.95	4480	6.94	7.00			
19	1790	8.36	8.41	6280	7.35	7.40			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.15$.

	r =10.00			r =15.00			r =25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	1.70	ACCEPT		0.680	ACCEPT		0.309	ACCEPT	
0	2.46	0.0000	0.0114	1.72	0.0000	0.0276	1.29	0.0000	0.0319
1	5.89	0.142	0.230	5.58	0.118	0.199	5.77	0.0912	0.154
2	13.2	0.394	0.475	15.6	0.316	0.386	21.8	0.229	0.285
3	27.9	0.652	0.727	40.9	0.511	0.577	78.3	0.365	0.418
4	57.3	0.909	0.980	105	0.705	0.768	273	0.499	0.551
5	116	1.17	1.23	263	0.899	0.961	936	0.634	0.684
6	230	1.42	1.49	653	1.09	1.15	3170	0.768	0.818
7	455	1.68	1.74	1610	1.29	1.35	10600	0.902	0.952
8	893	1.93	2.00	3930	1.48	1.54			
9	1740	2.19	2.25	9540	1.67	1.73			
10	3390	2.45	2.51	23100	1.87	1.93			
11	6570	2.70	2.77						
12	12700	2.96	3.02						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.25$.

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							17.7	ACCEPT	
3				24.7	ACCEPT		20.6	1.23	1.28
4	37.2	ACCEPT		28.1	1.82	1.87	25.1	1.82	1.88
5	38.4	2.47	2.48	32.9	2.46	2.51	30.4	2.42	2.49
6	43.0	3.12	3.17	38.2	3.10	3.15	36.4	3.03	3.09
7	48.1	3.81	3.86	44.2	3.75	3.80	43.2	3.64	3.70
8	53.8	4.50	4.55	50.8	4.40	4.44	50.8	4.25	4.31
9	59.9	5.19	5.24	58.0	5.04	5.09	59.4	4.87	4.92
10	66.4	5.89	5.93	66.0	5.69	5.74	69.1	5.48	5.52
11	73.5	6.58	6.62	74.7	6.34	6.38	79.9	6.09	6.13
12	81.1	7.27	7.31	84.3	6.99	7.03	92.1	6.70	6.74
13	89.2	7.96	8.00	94.7	7.64	7.68	106	7.31	7.35
14	97.9	8.66	8.69	106	8.29	8.33	121	7.92	7.96
15	107	9.35	9.38	119	8.94	8.97	138	8.53	8.57
16	117	10.0	10.1	133	9.58	9.62	158	9.14	9.18
17	128	10.7	10.8	148	10.2	10.3	180	9.75	9.79
18	139	11.4	11.5	164	10.9	10.9	204	10.4	10.4
19	151	12.1	12.2	182	11.5	11.6	232	11.0	11.0

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.25$.

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
							6.91	ACCEPT	
1	13.3	ACCEPT		10.5	ACCEPT		7.98	0.266	0.295
2	14.6	0.706	0.733	13.2	0.696	0.758	11.6	0.678	0.771
3	18.7	1.22	1.30	17.5	1.21	1.29	16.4	1.17	1.26
4	23.6	1.79	1.87	22.8	1.76	1.83	22.3	1.68	1.75
5	29.4	2.37	2.44	29.0	2.31	2.38	29.7	2.18	2.25
6	36.0	2.95	3.01	36.5	2.86	2.92	38.9	2.68	2.75
7	43.7	3.53	3.58	45.2	3.41	3.47	50.3	3.18	3.24
8	52.6	4.10	4.16	55.6	3.96	4.01	64.4	3.68	3.74
9	62.8	4.68	4.74	67.8	4.51	4.56	81.9	4.18	4.24
10	74.6	5.26	5.31	82.2	5.06	5.11	104	4.69	4.74
11	88.2	5.84	5.89	99.3	5.61	5.66	131	5.19	5.24
12	104	6.42	6.47	119	6.16	6.21	164	5.69	5.74
13	122	7.00	7.04	143	6.70	6.75	205	6.19	6.24
14	143	7.57	7.62	171	7.25	7.30	257	6.69	6.74
15	167	8.15	8.20	205	7.80	7.85	320	7.19	7.24
16	194	8.73	8.77	244	8.35	8.40	399	7.69	7.74
17	226	9.31	9.35	290	8.90	8.95	496	8.19	8.24
18	263	9.89	9.93	345	9.45	9.50	615	8.70	8.75
19	305	10.5	10.5	409	10.0	10.0	763	9.20	9.25

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.25$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				3.00	ACCEPT		1.20	ACCEPT	
0	4.99	ACCEPT		3.25	0.0000	0.0036	2.33	0.0000	0.0343
1	6.93	0.260	0.323	5.89	0.235	0.337	5.23	0.212	0.327
2	10.8	0.663	0.762	10.1	0.622	0.721	9.97	0.559	0.653
3	16.0	1.13	1.21	16.1	1.03	1.11	17.6	0.904	0.987
4	22.7	1.59	1.67	24.7	1.43	1.51	29.7	1.25	1.32
5	31.4	2.05	2.12	36.9	1.84	1.91	49.1	1.59	1.66
6	42.8	2.52	2.58	54.2	2.24	2.31	80.0	1.93	2.00
7	57.6	2.98	3.04	78.7	2.64	2.71	129	2.27	2.34
8	76.8	3.44	3.50	113	3.05	3.11	207	2.61	2.68
9	102	3.90	3.96	163	3.45	3.51	330	2.95	3.02
10	134	4.37	4.43	232	3.85	3.91	525	3.29	3.36
11	176	4.83	4.89	330	4.25	4.32	831	3.63	3.70
12	231	5.29	5.35	469	4.66	4.72	1310	3.97	4.04
13	302	5.75	5.81	663	5.06	5.12	2070	4.31	4.38
14	394	6.22	6.27	937	5.46	5.52	3250	4.65	4.72
15	512	6.68	6.73	1320	5.86	5.92	5090	4.99	5.06
16	666	7.14	7.19	1860	6.27	6.33	7960	5.33	5.40
17	864	7.60	7.66	2610	6.67	6.73	12400	5.68	5.74
18	1120	8.06	8.12	3670	7.07	7.13			
19	1450	8.53	8.58	5140	7.47	7.53			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.25$.

	r = 10.00			r = 15.00			r = 25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.500	ACCEPT		0.273	ACCEPT		0.143	ACCEPT	
0	1.66	0.0000	0.0548	1.33	0.0000	0.0567	1.07	0.0000	0.0495
1	4.74	0.181	0.283	4.66	0.150	0.233	4.95	0.111	0.174
2	10.9	0.448	0.530	13.1	0.351	0.421	18.8	0.250	0.305
3	23.2	0.708	0.782	34.7	0.547	0.612	67.7	0.386	0.438
4	47.8	0.965	1.04	88.9	0.742	0.804	236	0.520	0.572
5	96.5	1.22	1.29	224	0.935	0.997	811	0.655	0.705
6	193	1.48	1.54	556	1.13	1.19	2740	0.789	0.839
7	381	1.73	1.80	1370	1.32	1.38	9200	0.923	0.973
8	749	1.99	2.05	3350	1.52	1.58	30600	1.06	1.11
9	1460	2.25	2.31	8150	1.71	1.77			
10	2850	2.50	2.57	19700	1.90	1.96			
11	5520	2.76	2.82						
12	10700	3.01	3.08						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.40.$

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				8.42	ACCEPT		5.78	ACCEPT	
1	12.9	ACCEPT		8.87	0.344	0.358	7.45	0.351	0.419
2	14.2	0.874	0.906	11.8	0.877	0.975	10.6	0.888	0.998
3	17.4	1.50	1.58	15.3	1.52	1.61	14.2	1.50	1.59
4	21.1	2.19	2.26	19.1	2.16	2.24	18.2	2.11	2.19
5	25.0	2.88	2.95	23.4	2.81	2.89	22.7	2.73	2.80
6	29.3	3.57	3.64	28.0	3.46	3.53	27.7	3.34	3.40
7	33.8	4.27	4.32	33.0	4.11	4.17	33.3	3.95	4.01
8	38.6	4.96	5.01	38.4	4.76	4.82	39.6	4.56	4.62
9	43.8	5.66	5.70	44.4	5.41	5.46	46.7	5.17	5.23
10	49.3	6.35	6.39	50.9	6.06	6.11	54.5	5.79	5.84
11	55.1	7.04	7.09	58.1	6.71	6.76	63.4	6.40	6.45
12	61.3	7.74	7.78	65.9	7.36	7.41	73.3	7.01	7.05
13	68.0	8.43	8.47	74.4	8.01	8.05	84.5	7.62	7.66
14	75.1	9.12	9.16	83.7	8.66	8.70	97.0	8.23	8.27
15	82.7	9.82	9.85	93.9	9.31	9.35	111	8.84	8.88
16	90.8	10.5	10.5	105	9.96	10.0	127	9.45	9.49
17	99.4	11.2	11.2	117	10.6	10.6	145	10.1	10.1
18	109	11.9	11.9	131	11.3	11.3	164	10.7	10.7
19	118	12.6	12.6	145	11.9	11.9	187	11.3	11.3

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.40.$

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
				3.00	ACCEPT		1.50	ACCEPT	
0	4.43	ACCEPT		3.62	0.0000	0.0130	2.82	0.0000	0.0506
1	6.63	0.346	0.451	6.10	0.336	0.467	5.45	0.327	0.477
2	9.83	0.884	0.999	9.34	0.873	0.989	8.75	0.840	0.955
3	13.5	1.47	1.56	13.1	1.43	1.52	12.8	1.35	1.44
4	17.7	2.05	2.13	17.6	1.98	2.07	17.8	1.86	1.94
5	22.6	2.63	2.70	22.8	2.54	2.61	23.9	2.36	2.43
6	28.1	3.21	3.28	28.9	3.09	3.15	31.5	2.86	2.93
7	34.4	3.79	3.85	36.1	3.64	3.70	40.9	3.37	3.43
8	41.7	4.37	4.43	44.6	4.19	4.25	52.5	3.87	3.93
9	50.1	4.95	5.00	54.6	4.74	4.79	66.9	4.37	4.43
10	59.7	5.53	5.58	66.5	5.29	5.34	84.8	4.87	4.93
11	70.8	6.11	6.16	80.4	5.84	5.89	107	5.37	5.43
12	83.6	6.68	6.73	96.9	6.39	6.44	134	5.87	5.93
13	98.3	7.26	7.31	116	6.94	6.99	168	6.38	6.43
14	115	7.84	7.89	139	7.49	7.54	211	6.88	6.93
15	135	8.42	8.46	166	8.04	8.08	263	7.38	7.43
16	157	9.00	9.04	199	8.59	8.63	327	7.88	7.93
17	183	9.58	9.62	236	9.14	9.18	407	8.38	8.43
18	213	10.2	10.2	281	9.69	9.73	505	8.88	8.93
19	247	10.7	10.8	333	10.2	10.3	627	9.38	9.43

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.40$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	1.00	ACCEPT		0.600	ACCEPT		0.375	ACCEPT	
0	2.42	0.0000	0.0722	2.01	0.0000	0.0930	1.69	0.0000	0.102
1	5.05	0.322	0.473	4.60	0.306	0.449	4.27	0.280	0.408
2	8.44	0.802	0.914	8.21	0.730	0.835	8.31	0.639	0.736
3	12.8	1.27	1.37	13.3	1.14	1.23	14.7	0.987	1.07
4	18.4	1.74	1.82	20.4	1.55	1.63	25.0	1.33	1.41
5	25.7	2.21	2.28	30.6	1.95	2.03	41.3	1.67	1.74
6	35.1	2.67	2.74	45.0	2.36	2.43	67.3	2.01	2.08
7	47.3	3.13	3.20	65.5	2.76	2.83	109	2.36	2.42
8	63.2	3.60	3.66	94.5	3.16	3.23	174	2.70	2.76
9	83.9	4.06	4.12	136	3.57	3.63	278	3.04	3.10
10	111	4.52	4.58	194	3.97	4.03	442	3.38	3.44
11	146	4.98	5.04	275	4.37	4.43	700	3.72	3.78
12	191	5.45	5.50	391	4.77	4.83	1110	4.06	4.12
13	249	5.91	5.97	553	5.18	5.24	1740	4.40	4.46
14	325	6.37	6.43	781	5.58	5.64	2740	4.74	4.80
15	423	6.83	6.89	1100	5.98	6.04	4290	5.08	5.14
16	550	7.30	7.35	1550	6.38	6.44	6720	5.42	5.48
17	714	7.76	7.81	2180	6.79	6.84	10500	5.76	5.82
18	925	8.22	8.27	3060	7.19	7.25			
19	1200	8.68	8.74	4290	7.59	7.65			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.40$.

	r = 10.00			r = 15.00			r = 25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.200	ACCEPT		0.120	ACCEPT		0.0667	ACCEPT	
0	1.36	0.0000	0.0977	1.14	0.0000	0.0849	0.951	0.0000	0.0663
1	4.03	0.229	0.332	4.05	0.182	0.265	4.37	0.131	0.193
2	9.31	0.498	0.580	11.4	0.384	0.453	16.6	0.270	0.324
3	19.8	0.759	0.833	30.1	0.580	0.645	59.5	0.405	0.457
4	40.9	1.02	1.09	77.0	0.775	0.837	208	0.540	0.591
5	82.5	1.27	1.34	194	0.969	1.03	713	0.674	0.725
6	165	1.53	1.60	482	1.16	1.22	2410	0.809	0.858
7	326	1.79	1.85	1190	1.36	1.42	8090	0.943	0.992
8	641	2.04	2.11	2910	1.55	1.61	26900	1.08	1.13
9	1250	2.30	2.36	7070	1.74	1.80			
10	2440	2.55	2.62	17100	1.94	2.00			
11	4730	2.81	2.87						
12	9140	3.07	3.13						
13	17600	3.32	3.38						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.65.$

C	r = 2.00 M	SAMPLE SIZE		r = 2.25 M	SAMPLE SIZE		r = 2.50 M	SAMPLE SIZE	
	1.17	ACCEPT		0.757	ACCEPT		0.560	ACCEPT	
0	3.05	0.0000	0.126	2.65	0.0000	0.165	2.42	0.0000	0.187
1	5.82	0.513	0.726	5.31	0.526	0.736	4.98	0.526	0.730
2	8.70	1.24	1.39	8.12	1.21	1.36	7.75	1.18	1.31
3	11.7	1.95	2.06	11.1	1.88	1.99	10.8	1.80	1.91
4	14.9	2.65	2.74	14.4	2.54	2.63	14.2	2.42	2.51
5	18.3	3.35	3.43	17.9	3.19	3.27	17.9	3.04	3.12
6	21.8	4.05	4.12	21.7	3.84	3.91	22.1	3.65	3.72
7	25.6	4.74	4.81	25.9	4.49	4.56	26.7	4.27	4.33
8	29.6	5.44	5.50	30.4	5.15	5.20	31.9	4.88	4.94
9	33.8	6.13	6.19	35.3	5.80	5.85	37.7	5.49	5.55
10	38.3	6.83	6.88	40.6	6.45	6.50	44.1	6.11	6.16
11	43.1	7.52	7.57	46.5	7.10	7.14	51.4	6.72	6.77
12	48.2	8.22	8.26	52.8	7.75	7.79	59.5	7.33	7.38
13	53.6	8.91	8.95	59.8	8.40	8.44	68.6	7.94	7.99
14	59.4	9.60	9.64	67.4	9.05	9.09	78.8	8.55	8.60
15	65.6	10.3	10.3	75.7	9.69	9.74	90.3	9.16	9.21
16	72.2	11.0	11.0	84.8	10.3	10.4	103	9.77	9.82
17	79.2	11.7	11.7	94.8	11.0	11.0	118	10.4	10.4
18	86.7	12.4	12.4	106	11.6	11.7	134	11.0	11.0
19	94.7	13.1	13.1	118	12.3	12.3	152	11.6	11.6

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.65.$

C	r = 2.75 M	SAMPLE SIZE		r = 3.00 M	SAMPLE SIZE		r = 3.50 M	SAMPLE SIZE	
	0.444	ACCEPT		0.368	ACCEPT		0.275	ACCEPT	
0	2.25	0.0000	0.198	2.13	0.0000	0.204	1.94	0.0000	0.208
1	4.73	0.519	0.716	4.55	0.509	0.699	4.27	0.486	0.662
2	7.49	1.14	1.27	7.30	1.10	1.23	7.07	1.02	1.14
3	10.6	1.73	1.83	10.5	1.66	1.76	10.5	1.54	1.63
4	14.1	2.32	2.40	14.2	2.22	2.30	14.6	2.04	2.13
5	18.1	2.90	2.98	18.5	2.77	2.85	19.7	2.55	2.63
6	22.7	3.48	3.55	23.6	3.33	3.39	26.0	3.05	3.12
7	27.9	4.06	4.13	29.6	3.88	3.94	33.8	3.56	3.62
8	33.9	4.64	4.70	36.6	4.43	4.49	43.4	4.06	4.12
9	40.8	5.22	5.28	44.8	4.98	5.04	55.3	4.56	4.62
10	48.8	5.80	5.85	54.5	5.53	5.58	70.1	5.06	5.12
11	57.8	6.38	6.43	66.0	6.08	6.13	88.4	5.57	5.62
12	68.3	6.96	7.01	79.5	6.63	6.68	111	6.07	6.12
13	80.3	7.54	7.59	95.5	7.18	7.23	139	6.57	6.62
14	94.2	8.12	8.16	114	7.73	7.78	174	7.07	7.12
15	110	8.69	8.74	137	8.28	8.33	217	7.57	7.62
16	128	9.27	9.32	163	8.83	8.87	270	8.07	8.12
17	150	9.85	9.90	194	9.38	9.42	335	8.58	8.63
18	174	10.4	10.5	230	9.93	9.97	416	9.08	9.13
19	202	11.0	11.1	273	10.5	10.5	516	9.58	9.63

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.65$.

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.219	ACCEPT		0.156	ACCEPT		0.109	ACCEPT	
0	1.81	0.0000	0.205	1.62	0.0000	0.194	1.43	0.0000	0.177
1	4.08	0.461	0.626	3.84	0.416	0.564	3.65	0.362	0.491
2	6.95	0.954	1.07	6.90	0.845	0.952	7.09	0.724	0.821
3	10.6	1.43	1.52	11.2	1.26	1.35	12.5	1.07	1.16
4	15.3	1.90	1.98	17.2	1.67	1.75	21.2	1.42	1.49
5	21.4	2.36	2.44	25.7	2.07	2.14	35.0	1.76	1.83
6	29.2	2.83	2.90	37.8	2.48	2.55	57.0	2.10	2.17
7	39.4	3.29	3.36	54.9	2.88	2.95	91.9	2.44	2.51
8	52.6	3.76	3.82	79.1	3.28	3.35	147	2.78	2.85
9	69.7	4.22	4.28	113	3.69	3.75	235	3.12	3.19
10	91.9	4.68	4.74	162	4.09	4.15	373	3.47	3.53
11	121	5.15	5.20	230	4.49	4.55	590	3.81	3.87
12	158	5.61	5.66	326	4.89	4.95	931	4.15	4.21
13	207	6.07	6.13	462	5.30	5.36	1470	4.49	4.55
14	269	6.53	6.59	652	5.70	5.76	2300	4.83	4.89
15	350	6.99	7.05	919	6.10	6.16	3610	5.17	5.23
16	455	7.46	7.51	1290	6.50	6.56	5660	5.51	5.57
17	591	7.92	7.97	1820	6.91	6.97	8840	5.85	5.91
18	765	8.38	8.43	2550	7.31	7.37	13800	6.19	6.25
19	991	8.84	8.90	3580	7.71	7.77			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 0.65$.

	r = 10.00			r = 15.00			r = 25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.0636	ACCEPT		0.0400	ACCEPT		0.0230	ACCEPT	
0	1.20	0.0000	0.144	1.03	0.0000	0.115	0.866	0.0000	0.0839
1	3.51	0.281	0.383	3.57	0.215	0.297	3.90	0.151	0.212
2	8.06	0.551	0.632	9.98	0.418	0.487	14.7	0.290	0.344
3	17.1	0.812	0.885	26.2	0.615	0.678	52.5	0.426	0.477
4	35.1	1.07	1.14	66.9	0.809	0.871	183	0.560	0.611
5	70.8	1.33	1.39	168	1.00	1.06	627	0.695	0.744
6	141	1.58	1.65	418	1.20	1.26	2120	0.829	0.878
7	279	1.84	1.90	1030	1.39	1.45	7110	0.963	1.01
8	548	2.10	2.16	2520	1.58	1.64	23700	1.10	1.15
9	1070	2.35	2.42	6120	1.78	1.84			
10	2090	2.61	2.67	14800	1.97	2.03			
11	4040	2.86	2.93						
12	7810	3.12	3.18						
13	15100	3.38	3.44						

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 1.00$.

	r = 2.00			r = 2.25			r = 2.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	2.25	0.0000	0.493	2.12	0.0000	0.462	2.02	0.0000	0.435
1	4.52	0.899	1.14	4.32	0.841	1.07	4.16	0.792	1.00
2	6.91	1.64	1.81	6.68	1.54	1.69	6.51	1.45	1.59
3	9.45	2.36	2.49	9.24	2.21	2.33	9.11	2.08	2.19
4	12.1	3.07	3.17	12.0	2.87	2.97	12.0	2.70	2.80
5	15.0	3.77	3.86	15.0	3.53	3.61	15.2	3.32	3.40
6	18.0	4.47	4.55	18.2	4.18	4.26	18.7	3.94	4.01
7	21.2	5.17	5.23	21.8	4.83	4.90	22.7	4.55	4.62
8	24.5	5.86	5.92	25.6	5.49	5.55	27.1	5.16	5.22
9	28.1	6.56	6.62	29.8	6.14	6.19	32.0	5.78	5.83
10	31.9	7.25	7.31	34.3	6.79	6.84	37.5	6.39	6.44
11	36.0	7.95	8.00	39.2	7.44	7.49	43.6	7.00	7.05
12	40.3	8.64	8.69	44.6	8.09	8.14	50.5	7.61	7.66
13	44.9	9.34	9.38	50.5	8.74	8.78	58.2	8.23	8.27
14	49.8	10.0	10.1	56.9	9.39	9.43	66.8	8.84	8.88
15	55.0	10.7	10.8	63.9	10.0	10.1	76.4	9.45	9.49
16	60.6	11.4	11.5	71.6	10.7	10.7	87.3	10.1	10.1
17	66.5	12.1	12.2	80.0	11.3	11.4	99.4	10.7	10.7
18	72.8	12.8	12.8	89.1	12.0	12.0	113	11.3	11.3
19	79.5	13.5	13.5	99.1	12.6	12.7	128	11.9	11.9

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 1.00$.

	r = 2.75			r = 3.00			r = 3.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	1.94	0.0000	0.412	1.86	0.0000	0.392	1.73	0.0000	0.358
1	4.03	0.749	0.951	3.91	0.711	0.904	3.74	0.648	0.826
2	6.38	1.37	1.51	6.28	1.30	1.44	6.15	1.19	1.31
3	9.04	1.97	2.08	9.02	1.87	1.97	9.08	1.70	1.80
4	12.1	2.56	2.65	12.2	2.43	2.52	12.7	2.21	2.30
5	15.5	3.14	3.22	15.9	2.98	3.06	17.0	2.72	2.80
6	19.4	3.72	3.80	20.3	3.54	3.61	22.4	3.22	3.29
7	23.9	4.30	4.37	25.3	4.09	4.15	29.1	3.73	3.79
8	29.0	4.89	4.95	31.3	4.64	4.70	37.3	4.23	4.29
9	34.8	5.47	5.52	38.3	5.19	5.25	47.4	4.73	4.79
10	41.5	6.04	6.10	46.6	5.74	5.80	60.0	5.24	5.29
11	49.2	6.62	6.68	56.3	6.29	6.35	75.5	5.74	5.79
12	58.1	7.20	7.25	67.7	6.84	6.89	94.7	6.24	6.29
13	68.2	7.78	7.83	81.2	7.39	7.44	118	6.74	6.79
14	79.9	8.36	8.41	97.1	7.94	7.99	148	7.24	7.29
15	93.3	8.94	8.98	116	8.49	8.54	184	7.74	7.79
16	109	9.52	9.56	138	9.04	9.09	229	8.25	8.30
17	127	10.1	10.1	164	9.59	9.64	284	8.75	8.80
18	147	10.7	10.7	195	10.1	10.2	353	9.25	9.30
19	170	11.3	11.3	231	10.7	10.7	437	9.75	9.80

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 1.00.$

	r = 4.00			r = 5.00			r = 6.50		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	1.63	0.0000	0.331	1.48	0.0000	0.289	1.33	0.0000	0.246
1	3.61	0.557	0.763	3.43	0.519	0.666	3.28	0.438	0.565
2	6.09	1.09	1.21	6.10	0.949	1.06	6.30	0.800	0.897
3	9.25	1.57	1.66	9.82	1.36	1.45	11.1	1.15	1.23
4	13.3	2.04	2.12	15.0	1.77	1.85	18.6	1.49	1.57
5	18.5	2.51	2.58	22.4	2.18	2.25	30.5	1.84	1.91
6	25.2	2.97	3.04	32.8	2.58	2.65	49.6	2.18	2.25
7	33.9	3.43	3.50	47.5	2.99	3.05	79.7	2.52	2.59
8	45.2	3.90	3.96	68.2	3.39	3.45	127	2.86	2.93
9	59.8	4.36	4.42	97.6	3.79	3.86	203	3.20	3.27
10	78.8	4.82	4.88	139	4.20	4.26	322	3.54	3.61
11	103	5.29	5.35	197	4.60	4.66	509	3.88	3.95
12	135	5.75	5.81	280	5.00	5.06	804	4.22	4.29
13	176	6.21	6.27	396	5.40	5.46	1260	4.56	4.63
14	230	6.68	6.73	558	5.81	5.87	1990	4.91	4.97
15	298	7.14	7.19	787	6.21	6.27	3110	5.25	5.31
16	387	7.60	7.65	1110	6.61	6.67	4870	5.59	5.65
17	502	8.06	8.12	1550	7.01	7.07	7610	5.93	5.99
18	651	8.52	8.58	2180	7.42	7.47	11900	6.27	6.33
19	842	8.99	9.04	3060	7.82	7.88			

SINGLE SAMPLING TABLES FOR TWO-POINT PRIOR. $\gamma_2 = 1.00.$

	r = 10.00			r = 15.00			r = 25.00		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	1.12	0.0000	0.186	0.965	0.0000	0.142	0.814	0.0000	0.0999
1	3.18	0.327	0.428	3.24	0.246	0.326	3.55	0.169	0.229
2	7.20	0.598	0.679	8.94	0.449	0.517	13.2	0.308	0.361
3	15.2	0.859	0.932	23.3	0.645	0.708	47.1	0.443	0.495
4	30.9	1.12	1.19	59.4	0.840	0.901	164	0.578	0.628
5	62.2	1.37	1.44	149	1.03	1.09	562	0.712	0.762
6	124	1.63	1.70	369	1.23	1.29	1900	0.847	0.896
7	244	1.89	1.95	909	1.42	1.48	6360	0.981	1.03
8	479	2.14	2.21	2220	1.62	1.67	21100	1.12	1.16
9	937	2.40	2.46	5400	1.81	1.87			
10	1820	2.66	2.72	13100	2.00	2.06			
11	3530	2.91	2.97						
12	6820	3.17	3.23						
13	13100	3.42	3.49						

13. Tables of optimal sampling plans for the gamma prior distribution

Description and use of the tables.

The tables of sampling plans give c and m as functions of M, where c and m for given M minimize the function

$$R(c,m,M) = m\delta + (M - m)\, d(c,m) \tag{101}$$

with

$$\delta = \lambda_s - 1 + \int_0^1 (1 - \lambda) w(\lambda)\, d\lambda, \tag{102}$$

$$d(c,m) = \int_0^1 (1 - \lambda)(1 - B(c,m\lambda))\, w(\lambda)\, d\lambda + \int_1^\infty (\lambda - 1) B(c,m\lambda)\, w(\lambda)\, d\lambda, \tag{103}$$

$$w(\lambda)\, d\lambda = e^{-s\lambda/\bar{\lambda}} (s\lambda/\bar{\lambda})^{s-1} d(s\lambda/\bar{\lambda})/\Gamma(s) \tag{104}$$

and $(\lambda_s, \bar{\lambda}, s)$ are given constants.

Suppose that the mean occurrence rate of defects for a process (lot) is μ per inspection unit and that μ itself is a random variable having a gamma (prior) distribution, $w_0(\mu)$ say, with parameters $(\bar{\mu}, s)$. Let the three cost functions be $k_s(\mu) = S_1 + S_2\mu$, $k_a(\mu) = A_1 + A_2\mu$ and $k_r(\mu) = R_1 + R_2\mu$. The standardized regret then becomes

$$R_\mu(c,m,M) = m(\mu_s - \mu_0)$$
$$+ (M - m)\left\{\int_0^{\mu_r} (\mu_r - \mu)(1 - B(c,m\mu))\, w_0(\mu)\, d\mu + \int_{\mu_r}^\infty (\mu - \mu_r) B(c,m\mu)\, w_0(\mu)\, d\mu\right\}$$

where $\mu_r = (R_1 - A_1)/(A_2 - R_2)$,

$$\mu_0 = \mu_r - \int_0^{\mu_r} (\mu_r - \mu)\, w_0(\mu)\, d\mu$$

and

$$\mu_s = \{(S_1 - A_1) + (S_2 - R_2)\bar{\mu}\}/(A_2 - R_2).$$

By a change of scale we introduce $\lambda = \mu/\mu_r$, $m^* = m\mu_r$ and $M^* = M\mu_r$ which leads to the equation

$$R_\mu(c,m,M) = R(c,m^*,M^*) \tag{105}$$

with parameters $\bar{\lambda} = \bar{\mu}/\mu_r$, s and

$$\lambda_s = 1 + \frac{S_1 - R_1}{R_1 - A_1} + \frac{S_2 - R_2}{A_2 - R_2}\bar{\lambda}. \tag{106}$$

The optimum sampling plan (c,m) corresponding to the regret $R_\mu(c,m,M)$ may thus be found by reading (c,m^*) for the argument $M^* = M\mu_r$ from the table with parameters $(\lambda_s, \bar{\mu}/\mu_r, s)$, λ_s being defined in (106), and finding the sampling plan as $(c, m^*/\mu_r)$.

The tables may also be used for obtaining an approximation to the solution of the analogous binomial problem with regret function

$$R_b(c,n,N) = n(p_s - p_0) + (N-n)\left\{\int_0^{p_r} (p_r - p)(1 - B(c,n,p))f(p)\,dp + \int_{p_r}^1 (p - p_r)B(c,n,p)f(p)\,dp\right\},$$

where (p_r, p_0, p_s) are defined analogously to (μ_r, μ_0, μ_s) above and $f(p)$ denotes a beta distribution with parameters (s,t) so that $\bar{p} = s/(s+t)$. By the usual approximations we get

$$R_b(c,n,N) \simeq R(c, np_r, Np_r) \tag{107}$$

with parameters $\bar{\lambda} = \bar{p}/p_r$, s and λ_s defined by (106). Thus, the binomial problem is solved (approximately) by reading (c,m) from the table for the argument $M = Np_r$ and finding the sampling plan as $(c, m/p_r)$. The approximation is good for $p_r < 0.1$.

The tables contain solutions for $\lambda_s = 1$ only. For $\lambda_s > 1$ a good approximation to the solution is obtained by entering the tables with $M^* = M\delta_0/\delta$ as argument, where $\delta_0 = \int_0^1 (1-\lambda)w(\lambda)d\lambda$. Auxiliary tables of δ_0/δ have been given.

Tables of sampling plans are given for the following values of $\bar{\lambda}$ and s:

$$\bar{\lambda} = 0.10\ (0.05)\ 0.40\ (0.10)\ 1.00.$$

$$s = 0.1,\ 0.2,\ 0.3,\ 0.5,\ 0.7,\ 1.0,\ 2.0,\ 3.0,\ 5.0,\ 7.0,\ 10.0,\ 20.0.$$

Apart from seven cases giving rise to rather small tables the solution has been tabulated with the double limitation that $M < 10{,}000$ and $c \leqq 19$.

Each line of the table contains c, $M_{c+0.5}$, m_{cl} and m_{cu}. For $M_{c-0.5} < M \leqq M_{c+0.5}$ the optimum acceptance number is c, and the optimum values of m corresponding to $M_{c-0.5}$ and $M_{c+0.5}$ are m_{cl} and m_{cu}, respectively. Intermediate values of m may be obtained by linear interpolation with respect to $\sqrt{M}$.

If the first entry in the column headed sample size is "Accept" then the minimum of R is obtained for $m = 0$, i.e. by acceptance without inspection, and min $R = M(\bar{\lambda} - 1 + \delta_0)$ for $M < M^*$, where M^* represents the first entry in the M-column.

Example 6.

Suppose that batches of electric cable are inspected for defects in the insulation, each batch consisting of $M = 500$ yards of cable. Previous inspection results have shown that the mean occurrence rate of defects in a batch, μ say, may be described as a gamma distributed random variable with parameters $s = 1$ and $\bar{\mu} = 0.4$. The cost functions are $k_s(\mu) = 1$, $k_r(\mu) = k_s(\mu)$ and $k_a(\mu) = 2\mu$ cost units per yard. We then find the break-even quality to be $\mu_r = 0.5$ defects/yard.

In order to make use of the tables of optimum sampling plans we compute $\lambda = \bar{\mu}/\mu_r = 0.8$, $\lambda_s = 1$ and $M^* = M\mu_r = 250$. The table shows that for $229 < M^* \leqq 278$ the acceptance number is $c = 9$ and the sample size is between 9.21 and 9.30. Interpolation gives $m^* = 9.25$ so that the sample size becomes $m^*/\mu_r = 18.50$ yards.

Interpolation in the tables.

We note the following interpolation rules.

Interpolation with respect to $\bar{\lambda}$. Use linear interpolation on $\sqrt{M_{c+0.5}}$, m_{cl} and m_{cu} with respect to $1/\bar{\lambda}$.

Interpolation with respect to s. Use quadratic interpolation on $\sqrt{M_{c+0.5}}$ in the neighbourhood of the minimum and linear interpolation on $\sqrt{M_{c+0.5}}$ otherwise. In both cases use $\log s$ as argument. Use linear interpolation on m_{cl} and m_{cu} with respect to s.

Interpolation with respect to $\bar{\lambda}$ *and s.* Start by interpolating with respect to $\bar{\lambda}$ for the two neighbouring s-values and then use quadratic interpolation with respect to s.

Example 7. Interpolation with respect to $\bar{\lambda}$.

Suppose that we want to determine the plan corresponding to $M = 36$ for $\bar{\lambda} = 0.75$ and $s = 3$. Using the tabular values of $M_{c+0.5}$ for $\bar{\lambda} = 0.70$ and 0.80 linear interpolation on $\sqrt{M_{c+0.5}}$ with $1/\bar{\lambda}$ as argument leads to the following results:

	$M_{c+0.5}$		$M_{c+0.5}$	
c	$\bar{\lambda} = 0.70$, $1/\bar{\lambda} = 1.4286$	$\bar{\lambda} = 0.80$, $1/\bar{\lambda} = 1.2500$	$\bar{\lambda} = 0.75$, Interpolate	$1/\bar{\lambda} = 1.3333$, Exact
2	39.4	23.9	30.6	29.4
3	54.9	35.6	44.1	42.9

The optimum value of c for $M = 36$ is thus $c = 3$.

From the tables we find for $c = 3$

$\bar{\lambda}$	m_{cl}	m_{cu}	
0.70	2.17	2.34	
0.80	2.67	2.86	
0.75	2.44	2.62	Interpolation
0.75	2.44	2.62	Exact

Finally, linear interpolation to $\sqrt{M} = \sqrt{36} = 6$ between $(\sqrt{30.6}, 2.44)$ and $(\sqrt{44.1}, 2.62)$ gives $m = 2.52$ so that the result is $(m,c) = (2.52, 3)$ as compared to the exact solution (2.53, 3).

Example 8. Interpolation with respect to s.

Suppose that the plans for $s = 0.3$ had not been tabulated and that we want to find the optimum plan for $M = 25$ and $\bar{\lambda} = 0.6$ by interpolation from the tabulated plans for $s = 0.2$ and 0.5.

By linear interpolation on $\sqrt{M_{c+0.5}}$ with log$s$ as argument we find

c	$M_{c+0.5}$ $s = 0.2$	$s = 0.5$	Interpolate $s = 0.3$	Exact $s = 0.3$
0	15.7	9.44	12.7	12.1
1	54.4	29.8	42.6	40.4

It is obvious from the tables that $c = 1$.

From the tables we find for $c = 1$

s	m_{cl}	m_{cu}	
0.2	1.14	1.54	
0.5	0.951	1.35	
0.3	1.08	1.48	Interpolation
0.3	1.08	1.48	Exact

Finally, linear interpolation to $\sqrt{M} = \sqrt{25} = 5$ gives $m = 1.27$ such that the result is $(m,c) = (1.27, 1)$ as compared to the exact solution (1.29, 1).

Example 9. Interpolation with respect to s.

Suppose that tables for $s = 0.7$ were not available and that we want to determine the optimum plan for $M = 3600$ with $s = 0.7$ and $\bar{\lambda} = 0.3$ by interpolation.

From the tables for $\bar{\lambda} = 0.3$ we find for $c = 16$ and 17 that $M_{c+0.5}$ is decreasing from $s = 0.3$ to $s = 0.5$ and then increasing. Hence we shall use Aitken's method for quadratic interpolation. For $\bar{\lambda} = 0.3$ and $c = 16$ we find

s	$M_{c+0.5}$	$\log 0.7 - \log s$	$\sqrt{M_{c+0.5}}$			$M_{c+0.5}$
0.3	3720	0.3680	60.99	55.25		
0.5	3310	0.1461	57.53		59.64	3560
1.0	4310	– 0.1549	65.65	61.47		

as compared with the exact value $M_{c+0.5} = 3470$.

Analogously we find for $c = 17$ that $M = 3960$ as compared to the exact $M = 3870$. Hence, the optimal acceptance number for $M = 3600$ is $c = 17$.

By linear interpolation on s we find for $c = 17$

s	m_{cl}	m_{cu}	
0.5	16.3	16.4	
1.0	15.2	15.2	
0.7	15.9	15.9	Interpolate
0.7	15.9	15.9	Exact

such that the result is $(m,c) = (15.9, 17)$ which equals the exact result.

Use of the asymptotic expansions.

To make the asymptotic expansions given in Section 10 easier to use the most important coefficients have been tabulated. We shall summarize the procedure below.

Construction of a table of optimum sampling plans. For successive values of c, $c = 0,1,2,\ldots$, compute $m(c)$ and $m(c+0.5)$ from

$$m(c) = c + 0.5 + s - s/\bar{\lambda}. \tag{108}$$

Compute $M(c+0.5)$ from

$$\sqrt{M} = (m - k_2)/k_1 \tag{109}$$

with $m = m(c+0.5)$. Finally, determine m_{cl} from

$$m_{cl} \simeq m(c) - \{2m(c+1)\}^{-1} \tag{110}$$

and find m_{cu} by changing the sign of the correction term in the expression above.

Construction of isolated sampling plans. Determine $m(M) = k_1\sqrt{M} + k_2$ and then $c(M) = m(M) - 0.5 - s + s/\bar{\lambda}$. Round $c(M)$ to the nearest integer, c say, and determine $m(c)$ from (108) and $M(c)$ from (109) with $m = m(c)$. Finally, use

$$m \simeq m(c) + \{\sqrt{M} - \sqrt{M(c)}\}k_1/m(c+1)$$

to determine the sample size.

If some of the quantities turn out to be negative then use accept without inspection.

Example 10.

Suppose that we want to determine the sampling plan for $M = 200$ with $\bar{\lambda} = 0.75$, $s = 7$ and $\lambda_s = 1$ without using the tables of optimum plans. By means of the auxiliary tables we get $m(M) = 1.150 \times 14.14 - 8.64 = 7.62$ and $c(M) = 7.62 - 0.50 - 7.00 + 7.0/0.75 = 9.45$ such that the optimum acceptance number is $c = 9$. We then find $m(9) = 9 + 0.5 + 7.0 - 7.0/0.75 = 7.167$ and $\sqrt{M(9)} = (7.167 + 8.639)/1.150 = 13.74$ such that the approximation to the sample size becomes $m = 7.167 + (14.14 - 13.74) \times 1.150/8.167 = 7.22$.

If instead the problem had been to determine the sampling plan corresponding to $M = 50$ we would have found $m(M) = 1.150 \times 7.081 - 8.639 = -0.507$ which is negative. Hence, the optimum plan would be to accept without inspection.

Table of k_1 in the asymptotic relation between m and M. $m = k_1\sqrt{M} + k_2$.

Mean	s											
	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0
0.10	0.1452	0.1368	0.1133	0.0687								
0.15	0.1705	0.1871	0.1800	0.1468	0.1119	0.0706						
0.20	0.1847	0.2186	0.2263	0.2127	0.1865	0.1450						
0.25	0.1937	0.2399	0.2593	0.2648	0.2516	0.2203	0.1196					
0.30	0.2000	0.2552	0.2837	0.3059	0.3062	0.2892	0.2008	0.1273				
0.35	0.2046	0.2666	0.3025	0.3388	0.3516	0.3499	0.2869	0.2140	0.1092			
0.40	0.2082	0.2756	0.3173	0.3655	0.3895	0.4027	0.3718	0.3109	0.1986	0.1209		
0.45	0.2109	0.2827	0.3293	0.3877	0.4215	0.4484	0.4523	0.4113	0.3091	0.2209	0.1282	
0.50	0.2132	0.2885	0.3392	0.4062	0.4488	0.4883	0.5271	0.5105	0.4331	0.3484	0.2411	
0.55	0.2150	0.2934	0.3475	0.4220	0.4723	0.5231	0.5957	0.6059	0.5637	0.4957	0.3912	0.1551
0.60	0.2166	0.2975	0.3546	0.4356	0.4926	0.5537	0.6585	0.6962	0.6960	0.6553	0.5713	0.3135
0.65	0.2179	0.3010	0.3607	0.4473	0.5105	0.5808	0.7157	0.7808	0.8263	0.8205	0.7723	0.5419
0.70	0.2191	0.3041	0.3660	0.4577	0.5262	0.6049	0.7678	0.8595	0.9523	0.9864	0.9857	0.8340
0.75	0.2201	0.3067	0.3706	0.4668	0.5402	0.6265	0.8154	0.9325	1.0729	1.1496	1.2043	1.1768
0.80	0.2210	0.3091	0.3747	0.4749	0.5527	0.6459	0.8589	1.0002	1.1872	1.3078	1.4228	1.5546
0.85	0.2217	0.3112	0.3784	0.4822	0.5639	0.6634	0.8987	1.0628	1.2951	1.4595	1.6373	1.9528
0.90	0.2224	0.3130	0.3817	0.4887	0.5740	0.6793	0.9353	1.1209	1.3965	1.6041	1.8453	2.3592
0.95	0.2230	0.3147	0.3846	0.4946	0.5832	0.6938	0.9689	1.1747	1.4918	1.7412	2.0451	2.7645
1.00	0.2236	0.3162	0.3873	0.5000	0.5916	0.7071	1.0000	1.2247	1.5811	1.8708	2.2361	3.1623

Table of $-k_2$ in the asymptotic relation between m and M. $m = k_1\sqrt{M} - (-k_2)$.

Mean	s											
	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0
0.10	1.1108	2.5433	4.3808	9.2708								
0.15	0.7108	1.4989	2.4475	4.8264	7.8475	13.5833						
0.20	0.5317	1.0600	1.6683	3.1250	4.9017	8.1667						
0.25	0.4308	0.8233	1.2608	2.2708	3.4608	5.5833	15.5833					
0.30	0.3664	0.6767	1.0142	1.7708	2.6364	4.1389	10.9167	20.4167				
0.35	0.3217	0.5773	0.8502	1.4477	2.1142	3.2432	8.1276	14.7364	33.1276			
0.40	0.2890	0.5058	0.7340	1.2240	1.7590	2.6458	6.3333	11.1458	24.1458	41.6458		
0.45	0.2639	0.4520	0.6475	1.0610	1.5043	2.2253	5.1142	8.7500	18.2623	30.7623	55.1142	
0.50	0.2442	0.4100	0.5808	0.9375	1.3142	1.9167	4.2500	7.0833	14.2500	23.4167	40.9167	
0.55	0.2282	0.3764	0.5279	0.8411	1.1676	1.6825	3.6164	5.8850	11.4263	18.3065	31.1371	95.6618
0.60	0.2150	0.3489	0.4850	0.7639	1.0517	1.5000	3.1389	5.0000	9.3889	14.6667	24.2500	70.6389
0.65	0.2039	0.3260	0.4495	0.7008	0.9580	1.3545	2.7707	4.3319	7.8891	12.0261	19.3190	53.0518
0.70	0.1945	0.3066	0.4196	0.6484	0.8808	1.2364	2.4813	3.8180	6.7670	10.0833	15.7466	40.5935
0.75	0.1864	0.2900	0.3942	0.6042	0.8164	1.1389	2.2500	3.4167	5.9167	8.6389	13.1389	31.7500
0.80	0.1793	0.2756	0.3722	0.5664	0.7618	1.0573	2.0625	3.0990	5.2656	7.5573	11.2292	25.5000
0.85	0.1731	0.2631	0.3532	0.5338	0.7151	0.9882	1.9086	2.8446	4.7633	6.7442	9.8325	21.1387
0.90	0.1676	0.2520	0.3364	0.5054	0.6747	0.9290	1.7809	2.6389	4.3735	6.1327	8.8179	18.1698
0.95	0.1627	0.2421	0.3216	0.4804	0.6394	0.8779	1.6738	2.4711	4.0699	5.6743	8.0912	16.2375
1.00	0.1583	0.2333	0.3083	0.4583	0.6083	0.8333	1.5833	2.3333	3.8333	5.3333	7.5833	15.0833

Table of the rejectable part of the gamma prior distribution. The rejectable part is given in per cent.

	s											
Mean	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0
0.10	2.41	1.30	0.65	0.16								
0.15	4.25	3.21	2.20	0.98	0.43	0.13						
0.20	5.86	5.24	4.21	2.53	1.49	0.67						
0.25	7.24	7.18	6.34	4.55	3.17	1.83	0.30					
0.30	8.44	8.98	8.43	6.79	5.28	3.57	0.98	0.28				
0.35	9.50	10.62	10.43	9.10	7.63	5.74	2.21	0.88	0.15			
0.40	10.44	12.12	12.30	11.38	10.08	8.21	4.04	2.03	0.53	0.15		
0.45	11.29	13.50	14.05	13.60	12.55	10.84	6.39	3.80	1.40	0.53	0.13	
0.50	12.06	14.77	15.68	15.73	14.99	13.53	9.16	6.20	2.93	1.42	0.50	
0.55	12.76	15.94	17.20	17.75	17.35	16.23	12.22	9.12	5.20	3.03	1.39	0.12
0.60	13.40	17.02	18.62	19.67	19.63	18.89	15.46	12.47	8.21	5.51	3.10	0.51
0.65	14.00	18.03	19.95	21.48	21.81	21.47	18.79	16.10	11.87	8.86	5.83	1.59
0.70	14.56	18.97	21.19	23.20	23.88	23.97	22.15	19.92	16.04	13.01	9.65	3.85
0.75	15.07	19.85	22.36	24.82	25.86	26.36	25.48	23.81	20.56	17.81	14.49	7.72
0.80	15.56	20.68	23.46	26.36	27.73	28.65	28.73	27.71	25.30	23.05	20.14	13.36
0.85	16.02	21.46	24.50	27.81	29.51	30.84	31.88	31.54	30.11	28.55	26.36	20.59
0.90	16.45	22.20	25.49	29.18	31.21	32.92	34.92	35.28	34.89	34.13	32.86	28.99
0.95	16.86	22.89	26.42	30.49	32.81	34.90	37.83	38.88	39.56	39.64	39.40	37.99
1.00	17.24	23.56	27.30	31.73	34.34	36.79	40.60	42.32	44.05	44.97	45.79	47.03

Table of δ_0 for a gamma prior distribution.

Mean	s											
	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0
0.10	0.9170	0.9052	0.9019	0.9003								
0.15	0.8916	0.8684	0.8591	0.8527	0.8509	0.8502						
0.20	0.8721	0.8382	0.8225	0.8090	0.8040	0.8013						
0.25	0.8564	0.8132	0.7911	0.7699	0.7606	0.7546	0.7504					
0.30	0.8433	0.7919	0.7639	0.7350	0.7210	0.7107	0.7017	0.7003				
0.35	0.8321	0.7734	0.7402	0.7040	0.6851	0.6701	0.6545	0.6513	0.6501			
0.40	0.8224	0.7573	0.7193	0.6763	0.6527	0.6328	0.6094	0.6034	0.6006	0.6001		
0.45	0.8138	0.7429	0.7007	0.6514	0.6234	0.5988	0.5670	0.5574	0.5518	0.5505	0.5501	
0.50	0.8060	0.7301	0.6839	0.6289	0.5968	0.5677	0.5275	0.5136	0.5043	0.5016	0.5004	
0.55	0.7990	0.7184	0.6687	0.6085	0.5726	0.5393	0.4908	0.4726	0.4587	0.4539	0.4513	0.4501
0.60	0.7927	0.7078	0.6549	0.5899	0.5505	0.5133	0.4571	0.4344	0.4155	0.4080	0.4034	0.4003
0.65	0.7868	0.6980	0.6422	0.5729	0.5303	0.4896	0.4261	0.3991	0.3750	0.3646	0.3573	0.3511
0.70	0.7814	0.6890	0.6305	0.5572	0.5118	0.4678	0.3976	0.3667	0.3376	0.3240	0.3137	0.3032
0.75	0.7764	0.6807	0.6196	0.5427	0.4946	0.4477	0.3716	0.3370	0.3033	0.2867	0.2732	0.2574
0.80	0.7717	0.6729	0.6095	0.5293	0.4788	0.4292	0.3478	0.3099	0.2721	0.2527	0.2362	0.2148
0.85	0.7672	0.6656	0.6001	0.5168	0.4641	0.4121	0.3259	0.2853	0.2439	0.2220	0.2030	0.1762
0.90	0.7631	0.6588	0.5913	0.5051	0.4504	0.3963	0.3059	0.2629	0.2186	0.1947	0.1734	0.1423
0.95	0.7592	0.6524	0.5830	0.4942	0.4377	0.3816	0.2875	0.2425	0.1958	0.1704	0.1476	0.1132
1.00	0.7555	0.6463	0.5752	0.4839	0.4258	0.3679	0.2707	0.2240	0.1755	0.1490	0.1251	0.0888

Table of δ_0/δ for a gamma prior distribution, $\lambda_s = 1.5$.

Mean	s											
	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0
0.10	0.6471	0.6442	0.6433	0.6429								
0.15	0.6407	0.6346	0.6321	0.6304	0.6299	0.6297						
0.20	0.6356	0.6264	0.6219	0.6180	0.6166	0.6158						
0.25	0.6314	0.6192	0.6127	0.6063	0.6034	0.6015	0.6001					
0.30	0.6278	0.6130	0.6044	0.5951	0.5905	0.5870	0.5839	0.5834				
0.35	0.6247	0.6074	0.5969	0.5847	0.5781	0.5727	0.5669	0.5657	0.5653			
0.40	0.6219	0.6023	0.5899	0.5749	0.5662	0.5586	0.5493	0.5469	0.5457	0.5455		
0.45	0.6194	0.5977	0.5836	0.5657	0.5549	0.5449	0.5314	0.5271	0.5246	0.5240	0.5239	
0.50	0.6172	0.5935	0.5777	0.5571	0.5441	0.5317	0.5134	0.5067	0.5021	0.5008	0.5002	
0.55	0.6151	0.5896	0.5722	0.5489	0.5338	0.5189	0.4954	0.4859	0.4785	0.4758	0.4744	0.4737
0.60	0.6132	0.5860	0.5671	0.5412	0.5241	0.5066	0.4776	0.4649	0.4538	0.4493	0.4465	0.4446
0.65	0.6114	0.5827	0.5622	0.5340	0.5147	0.4947	0.4601	0.4439	0.4286	0.4217	0.4168	0.4126
0.70	0.6098	0.5795	0.5577	0.5271	0.5058	0.4833	0.4430	0.4231	0.4031	0.3932	0.3855	0.3775
0.75	0.6083	0.5765	0.5534	0.5205	0.4973	0.4724	0.4263	0.4026	0.3776	0.3644	0.3533	0.3399
0.80	0.6068	0.5737	0.5494	0.5142	0.4892	0.4619	0.4102	0.3827	0.3524	0.3357	0.3209	0.3005
0.85	0.6054	0.5710	0.5455	0.5083	0.4814	0.4518	0.3946	0.3633	0.3279	0.3075	0.2887	0.2606
0.90	0.6041	0.5685	0.5418	0.5025	0.4739	0.4421	0.3796	0.3446	0.3042	0.2802	0.2576	0.2215
0.95	0.6029	0.5661	0.5383	0.4971	0.4668	0.4328	0.3651	0.3266	0.2814	0.2542	0.2279	0.1846
1.00	0.6017	0.5638	0.5350	0.4918	0.4599	0.4239	0.3512	0.3094	0.2598	0.2296	0.2001	0.1509

Table of δ_0/δ for a gamma prior, $\lambda_s = 2.0$.

Mean	s											
	0.1	0.2	0.3	0.5	0.7	1.0	2.0	3.0	5.0	7.0	10.0	20.0
0.10	0.4783	0.4751	0.4742	0.4738								
0.15	0.4713	0.4648	0.4621	0.4602	0.4597	0.4595						
0.20	0.4658	0.4560	0.4513	0.4472	0.4457	0.4449						
0.25	0.4613	0.4485	0.4417	0.4350	0.4320	0.4301	0.4287					
0.30	0.4575	0.4419	0.4331	0.4236	0.4189	0.4154	0.4123	0.4119				
0.35	0.4542	0.4361	0.4254	0.4131	0.4066	0.4012	0.3956	0.3944	0.3940			
0.40	0.4513	0.4309	0.4184	0.4034	0.3949	0.3876	0.3787	0.3763	0.3752	0.3750		
0.45	0.4487	0.4263	0.4120	0.3944	0.3840	0.3745	0.3618	0.3579	0.3556	0.3551	0.3549	
0.50	0.4463	0.4220	0.4061	0.3861	0.3737	0.3621	0.3453	0.3393	0.3352	0.3340	0.3335	
0.55	0.4442	0.4181	0.4007	0.3783	0.3641	0.3503	0.3292	0.3209	0.3144	0.3122	0.3110	0.3104
0.60	0.4422	0.4145	0.3957	0.3710	0.3551	0.3392	0.3137	0.3028	0.2935	0.2898	0.2874	0.2859
0.65	0.4403	0.4111	0.3911	0.3642	0.3465	0.3287	0.2988	0.2852	0.2727	0.2672	0.2632	0.2599
0.70	0.4386	0.4079	0.3867	0.3578	0.3385	0.3187	0.2845	0.2683	0.2524	0.2447	0.2388	0.2327
0.75	0.4370	0.4050	0.3826	0.3518	0.3309	0.3092	0.2709	0.2521	0.2327	0.2228	0.2146	0.2047
0.80	0.4356	0.4022	0.3787	0.3461	0.3238	0.3003	0.2580	0.2366	0.2139	0.2017	0.1911	0.1768
0.85	0.4341	0.3996	0.3751	0.3407	0.3170	0.2918	0.2458	0.2220	0.1961	0.1817	0.1687	0.1498
0.90	0.4328	0.3972	0.3716	0.3356	0.3106	0.2838	0.2342	0.2082	0.1794	0.1630	0.1478	0.1246
0.95	0.4316	0.3948	0.3683	0.3307	0.3044	0.2762	0.2233	0.1952	0.1637	0.1456	0.1286	0.1017
1.00	0.4304	0.3926	0.3652	0.3261	0.2986	0.2689	0.2130	0.1830	0.1493	0.1297	0.1112	0.0816

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.10.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	90.0	ACCEPT							
0	94.6	0.0000	0.0040						
1	240	0.486	0.830						
2	495	1.48	1.77	1070	ACCEPT				
3	846	2.51	2.73	1390	1.72	1.85			
4	1290	3.53	3.71	1920	2.67	2.82			
5	1830	4.54	4.69	2580	3.67	3.81	6370	ACCEPT	
6	2470	5.55	5.68	3340	4.67	4.79	7110	3.84	3.90
7	3200	6.56	6.67	4220	5.67	5.78	8420	4.80	4.89
8	4030	7.56	7.67	5200	6.67	6.77	9920	5.80	5.88
9	4950	8.56	8.66	6290	7.68	7.77	11600	6.79	6.87
10	5970	9.57	9.65	7490	8.68	8.76			
11	7080	10.6	10.7	8800	9.68	9.76			
12	8280	11.6	11.6	10200	10.7	10.8			
13	9580	12.6	12.6						
14	11000	13.6	13.6						

$\bar{\lambda} = 0.15.$

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.15.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	8.72	ACCEPT							
0	44.1	0.0000	0.259	118	ACCEPT				
1	157	0.711	1.14	179	0.464	0.608	541	ACCEPT	
2	339	1.79	2.09	337	1.27	1.54	551	0.973	0.981
3	590	2.83	3.06	556	2.29	2.50	783	1.78	1.95
4	909	3.85	4.04	833	3.30	3.48	1090	2.76	2.92
5	1300	4.87	5.02	1170	4.31	4.46	1470	3.76	3.91
6	1750	5.88	6.01	1560	5.32	5.45	1920	4.77	4.89
7	2280	6.89	7.00	2010	6.33	6.44	2420	5.77	5.88
8	2880	7.89	8.00	2510	7.33	7.43	2990	6.77	6.87
9	3540	8.90	8.99	3080	8.33	8.43	3620	7.77	7.87
10	4270	9.90	9.99	3700	9.34	9.42	4310	8.78	8.86
11	5070	10.9	11.0	4370	10.3	10.4	5070	9.78	9.86
12	5940	11.9	12.0	5110	11.3	11.4	5880	10.8	10.9
13	6880	12.9	13.0	5900	12.3	12.4	6760	11.8	11.8
14	7890	13.9	14.0	6750	13.3	13.4	7700	12.8	12.8
15	8970	14.9	15.0	7660	14.3	14.4	8700	13.8	13.8
16	10100	15.9	16.0	8620	15.3	15.4	9770	14.8	14.8
17				9640	16.3	16.4	10900	15.8	15.8
18				10700	17.3	17.4			

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.20.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	3.33	ACCEPT		20.0	ACCEPT				
0	35.8	0.0000	0.398	37.5	0.0000	0.0746	105	ACCEPT	
1	131	0.873	1.30	108	0.528	0.921	138	0.449	0.541
2	286	1.96	2.25	223	1.57	1.86	247	1.22	1.48
3	499	3.00	3.22	380	2.61	2.83	400	2.23	2.44
4	770	4.02	4.20	579	3.63	3.81	593	3.24	3.41
5	1100	5.03	5.19	820	4.64	4.79	826	4.25	4.40
6	1490	6.04	6.18	1100	5.65	5.78	1100	5.26	5.39
7	1940	7.05	7.17	1430	6.65	6.77	1410	6.26	6.38
8	2440	8.06	8.16	1790	7.66	7.76	1760	7.26	7.37
9	3010	9.06	9.16	2200	8.66	8.76	2150	8.27	8.36
10	3630	10.1	10.2	2650	9.67	9.75	2580	9.27	9.36
11	4310	11.1	11.1	3140	10.7	10.7	3040	10.3	10.4
12	5050	12.1	12.1	3680	11.7	11.7	3550	11.3	11.3
13	5850	13.1	13.1	4250	12.7	12.7	4100	12.3	12.3
14	6710	14.1	14.1	4870	13.7	13.7	4680	13.3	13.3
15	7630	15.1	15.1	5530	14.7	14.7	5300	14.3	14.3
16	8600	16.1	16.1	6230	15.7	15.7	5970	15.3	15.3
17	9640	17.1	17.1	6980	16.7	16.7	6670	16.3	16.3
18	10700	18.1	18.1	7760	17.7	17.7	7410	17.3	17.3
19				8590	18.7	18.7	8190	18.3	18.3

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.20.

	S= 0.5			S= 0.7			
C	M	SAMPLE SIZE		M	SAMPLE SIZE		
	717	ACCEPT					
3	731	1.64	1.64				
4	950	2.49	2.62				
5	1230	3.48	3.61	2970	ACCEPT		
6	1560	4.48	4.59	2990	3.79	3.79	
7	1930	5.48	5.58	3480	4.71	4.79	
8	2350	6.48	6.57	4040	5.70	5.78	
9	2820	7.48	7.57	4680	6.70	6.77	
10	3330	8.48	8.56	5380	7.70	7.77	
11	3890	9.48	9.56	6140	8.69	8.76	
12	4490	10.5	10.6	6970	9.69	9.76	
13	5130	11.5	11.5	7850	10.7	10.8	
14	5820	12.5	12.5	8800	11.7	11.8	
15	6550	13.5	13.5	9800	12.7	12.7	
16	7330	14.5	14.5	10900	13.7	13.7	
17	8150	15.5	15.5				
18	9020	16.5	16.5				
19	9930	17.5	17.5				

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.25.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	1.71	ACCEPT		6.00	ACCEPT		36.0	ACCEPT	
0	32.3	0.0000	0.483	25.3	0.0000	0.230	36.1	0.0000	0.0003
1	119	0.973	1.39	84.9	0.685	1.11	85.0	0.503	0.825
2	259	2.06	2.34	179	1.76	2.05	169	1.48	1.77
3	453	3.09	3.32	309	2.80	3.02	284	2.51	2.73
4	700	4.12	4.30	473	3.82	4.00	428	3.53	3.71
5	1000	5.13	5.28	672	4.84	4.99	603	4.54	4.69
6	1350	6.14	6.27	905	5.85	5.98	807	5.55	5.68
7	1760	7.15	7.27	1170	6.85	6.97	1040	6.56	6.67
8	2220	8.16	8.26	1480	7.86	7.96	1300	7.56	7.67
9	2730	9.16	9.25	1810	8.86	8.96	1600	8.57	8.66
10	3300	10.2	10.2	2190	9.87	9.95	1920	9.57	9.65
11	3920	11.2	11.2	2590	10.9	10.9	2270	10.6	10.7
12	4590	12.2	12.2	3040	11.9	11.9	2660	11.6	11.6
13	5320	13.2	13.2	3510	12.9	12.9	3070	12.6	12.6
14	6100	14.2	14.2	4030	13.9	13.9	3510	13.6	13.6
15	6930	15.2	15.2	4570	14.9	14.9	3980	14.6	14.6
16	7820	16.2	16.2	5150	15.9	15.9	4480	15.6	15.6
17	8760	17.2	17.2	5770	16.9	16.9	5020	16.6	16.6
18	9750	18.2	18.2	6420	17.9	17.9	5580	17.6	17.6
19	10800	19.2	19.2	7100	18.9	18.9	6170	18.6	18.6

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.25.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	182	ACCEPT							
2	234	1.07	1.18						
3	348	1.96	2.14	642	ACCEPT				
4	496	2.95	3.12	756	2.44	2.52			
5	673	3.96	4.10	956	3.39	3.51			
6	879	4.96	5.09	1190	4.38	4.49	2860	ACCEPT	
7	1110	5.97	6.08	1470	5.38	5.48	2890	4.58	4.58
8	1380	6.97	7.07	1780	6.38	6.48	3290	5.51	5.58
9	1670	7.97	8.06	2120	7.38	7.47	3750	6.50	6.57
10	1990	8.97	9.06	2490	8.38	8.46	4260	7.50	7.57
11	2340	9.98	10.1	2890	9.38	9.46	4810	8.50	8.56
12	2720	11.0	11.1	3330	10.4	10.5	5420	9.50	9.56
13	3130	12.0	12.0	3800	11.4	11.5	6060	10.5	10.6
14	3560	13.0	13.0	4300	12.4	12.4	6760	11.5	11.6
15	4030	14.0	14.0	4830	13.4	13.4	7490	12.5	12.5
16	4520	15.0	15.0	5390	14.4	14.4	8270	13.5	13.5
17	5040	16.0	16.0	5990	15.4	15.4	9090	14.5	14.5
18	5590	17.0	17.0	6610	16.4	16.4	9950	15.5	15.5
19	6170	18.0	18.0	7270	17.4	17.4	10900	16.5	16.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.30.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	1.01	ACCEPT		2.92	ACCEPT		7.78	ACCEPT	
0	30.4	0.0000	0.541	21.2	0.0000	0.339	21.6	0.0000	0.149
1	112	1.04	1.45	73.6	0.812	1.23	65.9	0.604	1.01
2	244	2.12	2.41	157	1.89	2.18	136	1.67	1.96
3	425	3.16	3.38	271	2.93	3.15	230	2.70	2.93
4	657	4.18	4.36	415	3.95	4.13	350	3.72	3.90
5	938	5.20	5.35	591	4.97	5.12	494	4.74	4.89
6	1270	6.21	6.34	797	5.98	6.11	664	5.75	5.88
7	1650	7.22	7.33	1030	6.98	7.10	858	6.75	6.87
8	2080	8.22	8.33	1300	7.99	8.09	1080	7.76	7.86
9	2560	9.23	9.32	1600	8.99	9.09	1320	8.76	8.86
10	3100	10.2	10.3	1930	10.0	10.1	1590	9.77	9.85
11	3680	11.2	11.3	2290	11.0	11.1	1880	10.8	10.8
12	4310	12.2	12.3	2680	12.0	12.1	2200	11.8	11.8
13	4990	13.2	13.3	3100	13.0	13.1	2540	12.8	12.8
14	5720	14.2	14.3	3550	14.0	14.1	2910	13.8	13.8
15	6500	15.2	15.3	4030	15.0	15.1	3310	14.8	14.8
16	7340	16.2	16.3	4550	16.0	16.1	3720	15.8	15.8
17	8220	17.2	17.3	5090	17.0	17.1	4170	16.8	16.8
18	9150	18.2	18.3	5670	18.0	18.1	4630	17.8	17.8
19	10100	19.2	19.3	6270	19.0	19.1	5130	18.8	18.8

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.30.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	60.7	ACCEPT							
1	81.6	0.466	0.569	198	ACCEPT				
2	145	1.25	1.51	210	1.02	1.04			
3	231	2.26	2.47	296	1.84	2.01			
4	340	3.27	3.45	409	2.83	2.99	746	ACCEPT	
5	470	4.28	4.43	545	3.83	3.97	862	3.20	3.27
6	621	5.29	5.42	703	4.83	4.96	1050	4.16	4.26
7	794	6.29	6.41	884	5.84	5.95	1260	5.16	5.25
8	989	7.30	7.40	1090	6.84	6.94	1500	6.16	6.24
9	1200	8.30	8.39	1310	7.84	7.93	1760	7.15	7.24
10	1440	9.30	9.39	1550	8.84	8.93	2050	8.15	8.23
11	1700	10.3	10.4	1820	9.85	9.92	2370	9.15	9.23
12	1980	11.3	11.4	2110	10.8	10.9	2710	10.2	10.2
13	2280	12.3	12.4	2420	11.8	11.9	3070	11.2	11.2
14	2600	13.3	13.4	2750	12.8	12.9	3460	12.2	12.2
15	2950	14.3	14.4	3100	13.9	13.9	3870	13.2	13.2
16	3310	15.3	15.4	3470	14.9	14.9	4310	14.2	14.2
17	3700	16.3	16.4	3870	15.9	15.9	4770	15.2	15.2
18	4110	17.3	17.4	4280	16.9	16.9	5250	16.2	16.2
19	4540	18.3	18.4	4720	17.9	17.9	5760	17.2	17.2

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.35.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.652	ACCEPT		1.69	ACCEPT		3.59	ACCEPT	
0	29.3	0.0000	0.582	19.1	0.0000	0.419	17.2	0.0000	0.262
1	107	1.09	1.50	67.1	0.906	1.32	56.2	0.729	1.15
2	233	2.17	2.45	143	1.99	2.27	117	1.81	2.09
3	407	3.21	3.43	247	3.03	3.25	200	2.84	3.06
4	628	4.23	4.41	380	4.05	4.23	305	3.86	4.04
5	897	5.24	5.40	540	5.06	5.21	432	4.88	5.03
6	1210	6.25	6.39	729	6.07	6.20	580	5.89	6.02
7	1580	7.26	7.38	946	7.08	7.19	750	6.90	7.01
8	1990	8.27	8.37	1190	8.08	8.19	943	7.90	8.00
9	2450	9.27	9.37	1460	9.09	9.18	1160	8.90	9.00
10	2960	10.3	10.4	1760	10.1	10.2	1390	9.91	9.99
11	3520	11.3	11.4	2090	11.1	11.2	1650	10.9	11.0
12	4120	12.3	12.4	2450	12.1	12.2	1930	11.9	12.0
13	4770	13.3	13.4	2840	13.1	13.2	2230	12.9	13.0
14	5470	14.3	14.4	3250	14.1	14.2	2560	13.9	14.0
15	6220	15.3	15.3	3690	15.1	15.2	2900	14.9	15.0
16	7010	16.3	16.3	4160	16.1	16.2	3270	15.9	16.0
17	7850	17.3	17.3	4660	17.1	17.2	3660	16.9	17.0
18	8740	18.3	18.3	5190	18.1	18.2	4070	17.9	18.0
19	9680	19.3	19.3	5740	19.1	19.2	4500	18.9	19.0

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.35.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	26.9	ACCEPT		76.4	ACCEPT				
1	56.4	0.514	0.795	78.5	0.422	0.431			
2	108	1.46	1.74	126	1.16	1.37	252	ACCEPT	
3	178	2.49	2.70	194	2.14	2.34	282	1.73	1.78
4	265	3.50	3.68	279	3.15	3.31	369	2.63	2.76
5	370	4.51	4.66	381	4.15	4.30	477	3.62	3.75
6	492	5.52	5.65	499	5.16	5.29	603	4.62	4.73
7	632	6.53	6.64	633	6.16	6.28	747	5.62	5.72
8	789	7.53	7.64	783	7.17	7.27	907	6.62	6.72
9	964	8.54	8.63	950	8.17	8.26	1080	7.62	7.71
10	1160	9.54	9.63	1130	9.17	9.26	1280	8.62	8.70
11	1370	10.5	10.6	1330	10.2	10.3	1490	9.62	9.70
12	1590	11.5	11.6	1550	11.2	11.2	1710	10.6	10.7
13	1840	12.5	12.6	1780	12.2	12.2	1960	11.6	11.7
14	2100	13.5	13.6	2030	13.2	13.2	2210	12.6	12.7
15	2380	14.6	14.6	2290	14.2	14.2	2490	13.6	13.7
16	2670	15.6	15.6	2570	15.2	15.2	2780	14.6	14.7
17	2990	16.6	16.6	2870	16.2	16.2	3090	15.6	15.7
18	3320	17.6	17.6	3180	17.2	17.2	3420	16.6	16.7
19	3670	18.6	18.6	3510	18.2	18.2	3760	17.6	17.7

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.40.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.441	ACCEPT		1.07	ACCEPT		2.05	ACCEPT	
0	28.5	0.0000	0.613	17.9	0.0000	0.480	15.1	0.0000	0.351
1	104	1.12	1.53	62.8	0.977	1.39	50.5	0.832	1.25
2	226	2.20	2.49	134	2.06	2.34	106	1.91	2.20
3	394	3.24	3.46	232	3.10	3.32	181	2.95	3.17
4	608	4.27	4.44	356	4.12	4.30	276	3.97	4.15
5	868	5.28	5.43	506	5.13	5.28	391	4.98	5.14
6	1170	6.29	6.42	682	6.14	6.27	526	5.99	6.13
7	1530	7.30	7.41	885	7.15	7.27	680	7.00	7.12
8	1930	8.30	8.41	1110	8.16	8.26	855	8.01	8.11
9	2370	9.31	9.40	1370	9.16	9.25	1050	9.01	9.11
10	2860	10.3	10.4	1650	10.2	10.2	1260	10.0	10.1
11	3400	11.3	11.4	1960	11.2	11.2	1500	11.0	11.1
12	3980	12.3	12.4	2300	12.2	12.2	1750	12.0	12.1
13	4610	13.3	13.4	2660	13.2	13.2	2030	13.0	13.1
14	5290	14.3	14.4	3040	14.2	14.2	2320	14.0	14.1
15	6010	15.3	15.4	3460	15.2	15.2	2630	15.0	15.1
16	6780	16.3	16.4	3900	16.2	16.2	2970	16.0	16.1
17	7590	17.3	17.4	4360	17.2	17.2	3320	17.0	17.1
18	8450	18.3	18.4	4860	18.2	18.2	3690	18.0	18.1
19	9360	19.3	19.4	5380	19.2	19.2	4090	19.0	19.1

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.40.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	7.50	ACCEPT							
0	16.4	0.0000	0.107	31.6	ACCEPT				
1	45.0	0.577	0.962	50.8	0.503	0.677	100	ACCEPT	
2	89.3	1.63	1.91	91.8	1.36	1.62	121	1.09	1.17
3	149	2.66	2.88	147	2.37	2.58	174	1.96	2.14
4	223	3.68	3.85	215	3.39	3.56	242	2.96	3.12
5	313	4.69	4.84	297	4.40	4.54	323	3.96	4.10
6	417	5.70	5.83	392	5.40	5.53	417	4.97	5.09
7	536	6.71	6.82	500	6.41	6.52	523	5.97	6.08
8	671	7.71	7.81	621	7.41	7.52	642	6.97	7.07
9	820	8.71	8.81	756	8.42	8.51	773	7.97	8.06
10	984	9.72	9.80	904	9.42	9.51	916	8.98	9.06
11	1160	10.7	10.8	1060	10.4	10.5	1070	9.98	10.1
12	1360	11.7	11.8	1240	11.4	11.5	1240	11.0	11.0
13	1570	12.7	12.8	1430	12.4	12.5	1420	12.0	12.0
14	1790	13.7	13.8	1630	13.4	13.5	1610	13.0	13.0
15	2030	14.7	14.8	1840	14.4	14.5	1820	14.0	14.0
16	2280	15.7	15.8	2070	15.4	15.5	2040	15.0	15.0
17	2550	16.7	16.8	2310	16.4	16.5	2270	16.0	16.0
18	2840	17.7	17.8	2560	17.4	17.5	2510	17.0	17.0
19	3140	18.7	18.8	2830	18.4	18.5	2760	18.0	18.0

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.50.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.222	ACCEPT		0.500	ACCEPT		0.857	ACCEPT	
0	27.4	0.0000	0.657	16.4	0.0000	0.567	13.0	0.0000	0.478
1	99.7	1.18	1.58	57.5	1.08	1.48	44.0	0.979	1.39
2	216	2.25	2.54	123	2.16	2.44	92.3	2.06	2.34
3	376	3.29	3.51	212	3.19	3.41	158	3.10	3.31
4	581	4.31	4.49	325	4.22	4.39	241	4.12	4.30
5	829	5.33	5.48	462	5.23	5.38	342	5.13	5.28
6	1120	6.34	6.47	623	6.24	6.37	459	6.14	6.27
7	1460	7.35	7.46	808	7.25	7.36	595	7.15	7.27
8	1840	8.35	8.46	1020	8.25	8.36	747	8.16	8.26
9	2260	9.36	9.45	1250	9.26	9.35	917	9.16	9.25
10	2730	10.4	10.4	1510	10.3	10.3	1100	10.2	10.2
11	3240	11.4	11.4	1790	11.3	11.3	1310	11.2	11.2
12	3800	12.4	12.4	2090	12.3	12.3	1530	12.2	12.2
13	4400	13.4	13.4	2420	13.3	13.3	1770	13.2	13.2
14	5040	14.4	14.4	2780	14.3	14.3	2030	14.2	14.2
15	5730	15.4	15.4	3150	15.3	15.3	2300	15.2	15.2
16	6470	16.4	16.4	3560	16.3	16.3	2590	16.2	16.2
17	7240	17.4	17.4	3980	17.3	17.3	2900	17.2	17.2
18	8060	18.4	18.4	4430	18.3	18.3	3230	18.2	18.2
19	8930	19.4	19.4	4900	19.3	19.3	3570	19.2	19.2

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.50.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	2.00	ACCEPT		4.67	ACCEPT				
0	11.1	0.0000	0.305	11.9	0.0000	0.143	21.3	ACCEPT	
1	34.6	0.790	1.20	32.5	0.621	1.01	35.8	0.522	0.720
2	70.1	1.87	2.15	63.3	1.68	1.95	63.7	1.41	1.66
3	118	2.90	3.12	104	2.71	2.92	101	2.42	2.63
4	178	3.92	4.10	155	3.73	3.90	146	3.44	3.61
5	250	4.94	5.09	216	4.74	4.89	200	4.45	4.59
6	334	5.95	6.08	286	5.75	5.88	262	5.45	5.58
7	430	6.95	7.07	367	6.75	6.87	332	6.46	6.57
8	538	7.96	8.06	457	7.76	7.86	411	7.46	7.56
9	659	8.96	9.06	558	8.76	8.86	499	8.47	8.56
10	791	9.97	10.1	668	9.77	9.85	594	9.47	9.55
11	936	11.0	11.0	788	10.8	10.8	699	10.5	10.6
12	1090	12.0	12.0	918	11.8	11.8	811	11.5	11.5
13	1260	13.0	13.0	1060	12.8	12.8	932	12.5	12.5
14	1440	14.0	14.0	1210	13.8	13.8	1060	13.5	13.5
15	1640	15.0	15.0	1370	14.8	14.8	1200	14.5	14.5
16	1840	16.0	16.0	1540	15.8	15.8	1350	15.5	15.5
17	2060	17.0	17.0	1720	16.8	16.8	1500	16.5	16.5
18	2290	18.0	18.0	1910	17.8	17.8	1660	17.5	17.5
19	2530	19.0	19.0	2110	18.8	18.8	1830	18.5	18.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.50.

	S= 2.0			S= 3.0			
C	M	SAMPLE SIZE		M	SAMPLE SIZE		
	188	ACCEPT					
4	217	2.54	2.61				
5	268	3.49	3.60				
6	328	4.49	4.59				
7	397	5.48	5.58	771	ACCEPT		
8	473	6.48	6.57	796	5.56	5.57	
9	556	7.48	7.56	887	6.51	6.57	
10	647	8.48	8.56	990	7.50	7.56	
11	746	9.48	9.56	1100	8.50	8.56	
12	851	10.5	10.6	1220	9.50	9.55	
13	964	11.5	11.5	1350	10.5	10.6	
14	1080	12.5	12.5	1490	11.5	11.5	
15	1210	13.5	13.5	1640	12.5	12.5	
16	1350	14.5	14.5	1800	13.5	13.5	
17	1490	15.5	15.5	1960	14.5	14.5	
18	1640	16.5	16.5	2130	15.5	15.5	
19	1790	17.5	17.5	2310	16.5	16.5	

$\bar{\lambda} = 0.60.$

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.60.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.119	ACCEPT		0.256	ACCEPT		0.417	ACCEPT	
0	26.8	0.0000	0.687	15.7	0.0000	0.625	12.1	0.0000	0.564
1	96.9	1.21	1.61	54.4	1.14	1.54	40.4	1.08	1.48
2	210	2.29	2.57	116	2.22	2.50	84.7	2.16	2.44
3	365	3.33	3.54	200	3.26	3.48	145	3.20	3.41
4	563	4.35	4.53	306	4.28	4.46	221	4.22	4.39
5	804	5.36	5.51	435	5.30	5.45	313	5.23	5.38
6	1090	6.37	6.50	587	6.31	6.44	421	6.24	6.37
7	1410	7.38	7.50	761	7.31	7.43	545	7.25	7.36
8	1780	8.39	8.49	958	8.32	8.42	685	8.25	8.36
9	2190	9.39	9.48	1180	9.33	9.42	840	9.26	9.35
10	2650	10.4	10.5	1420	10.3	10.4	1010	10.3	10.3
11	3140	11.4	11.5	1680	11.3	11.4	1200	11.3	11.3
12	3680	12.4	12.5	1970	12.3	12.4	1400	12.3	12.3
13	4260	13.4	13.5	2280	13.3	13.4	1620	13.3	13.3
14	4890	14.4	14.5	2610	14.3	14.4	1860	14.3	14.3
15	5560	15.4	15.5	2970	15.3	15.4	2110	15.3	15.3
16	6270	16.4	16.5	3350	16.3	16.4	2380	16.3	16.3
17	7020	17.4	17.5	3750	17.3	17.4	2660	17.3	17.3
18	7810	18.4	18.5	4170	18.3	18.4	2960	18.3	18.3
19	8650	19.4	19.5	4620	19.3	19.4	3270	19.3	19.3

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.60.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.833	ACCEPT		1.46	ACCEPT		3.33	ACCEPT	
0	9.44	0.0000	0.445	8.70	0.0000	0.330	9.24	0.0000	0.165
1	29.8	0.951	1.35	25.9	0.827	1.22	24.2	0.656	1.03
2	60.7	2.03	2.31	51.3	1.90	2.18	45.9	1.71	1.98
3	102	3.07	3.28	85.0	2.94	3.15	74.2	2.74	2.95
4	154	4.09	4.26	127	3.96	4.13	109	3.76	3.93
5	217	5.10	5.25	177	4.97	5.12	150	4.77	4.92
6	290	6.11	6.24	235	5.98	6.11	198	5.78	5.91
7	373	7.12	7.23	302	6.99	7.10	252	6.79	6.90
8	467	8.12	8.23	377	7.99	8.09	313	7.79	7.89
9	572	9.13	9.22	460	9.00	9.09	381	8.80	8.89
10	687	10.1	10.2	551	10.0	10.1	455	9.80	9.88
11	813	11.1	11.2	651	11.0	11.1	535	10.8	10.9
12	949	12.1	12.2	759	12.0	12.1	622	11.8	11.9
13	1100	13.1	13.2	875	13.0	13.1	716	12.8	12.9
14	1250	14.1	14.2	999	14.0	14.1	816	13.8	13.9
15	1420	15.1	15.2	1130	15.0	15.1	922	14.8	14.9
16	1600	16.1	16.2	1270	16.0	16.1	1040	15.8	15.9
17	1790	17.1	17.2	1420	17.0	17.1	1150	16.8	16.9
18	1990	18.1	18.2	1580	18.0	18.1	1280	17.8	17.9
19	2200	19.1	19.2	1740	19.0	19.1	1410	18.8	18.9

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.60.

	S= 2.0			S= 3.0			S= 5.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	40.2	ACCEPT							
2	54.5	1.19	1.33						
3	77.8	2.12	2.30	136	ACCEPT				
4	106	3.12	3.27	149	2.56	2.61			
5	140	4.13	4.26	180	3.50	3.59			
6	178	5.13	5.25	218	4.49	4.58			
7	221	6.13	6.24	259	5.49	5.58			
8	268	7.14	7.23	306	6.49	6.57			
9	320	8.14	8.23	357	7.49	7.56			
10	377	9.14	9.22	412	8.49	8.56	726	ACCEPT	
11	438	10.1	10.2	471	9.49	9.55	789	8.18	8.22
12	504	11.1	11.2	535	10.5	10.6	858	9.18	9.22
13	575	12.1	12.2	602	11.5	11.5	932	10.2	10.2
14	650	13.1	13.2	674	12.5	12.5	1010	11.2	11.2
15	730	14.1	14.2	750	13.5	13.5	1100	12.2	12.2
16	814	15.2	15.2	831	14.5	14.5	1190	13.2	13.2
17	903	16.2	16.2	915	15.5	15.5	1280	14.2	14.2
18	997	17.2	17.2	1000	16.5	16.5	1380	15.2	15.2
19	1090	18.2	18.2	1100	17.5	17.5	1480	16.2	16.2

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.70.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.0640	ACCEPT		0.134	ACCEPT		0.211	ACCEPT	
0	26.3	0.0000	0.708	15.2	0.0000	0.667	11.5	0.0000	0.627
1	95.0	1.23	1.63	52.3	1.19	1.59	38.2	1.15	1.55
2	205	2.31	2.59	111	2.27	2.55	79.9	2.23	2.51
3	3[illegible]8	3.35	3.57	192	3.31	3.52	136	3.27	3.48
4	551	4.37	4.55	294	4.33	4.51	208	4.29	4.46
5	787	5.39	5.54	418	5.34	5.49	295	5.30	5.45
6	1060	6.40	6.53	563	6.35	6.48	396	6.31	6.44
7	1380	7.40	7.52	730	7.36	7.48	512	7.32	7.43
8	1740	8.41	8.51	919	8.37	8.47	644	8.33	8.43
9	2150	9.42	9.51	1130	9.37	9.47	790	9.33	9.42
10	2590	10.4	10.5	1360	10.4	10.5	951	10.3	10.4
11	3080	11.4	11.5	1610	11.4	11.5	1130	11.3	11.4
12	3600	12.4	12.5	1890	12.4	12.5	1320	12.3	12.4
13	4170	13.4	13.5	2190	13.4	13.5	1520	13.3	13.4
14	4780	14.4	14.5	2500	14.4	14.5	1740	14.3	14.4
15	5430	15.4	15.5	2840	15.4	15.4	1980	15.3	15.4
16	6130	16.4	16.5	3210	16.4	16.4	2230	16.3	16.4
17	6860	17.4	17.5	3590	17.4	17.4	2500	17.3	17.4
18	7640	18.4	18.5	3990	18.4	18.4	2780	18.3	18.4
19	8460	19.4	19.5	4420	19.4	19.4	3070	19.4	19.4

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.70.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.390	ACCEPT		0.612	ACCEPT		1.07	ACCEPT	
0	8.63	0.0000	0.548	7.53	0.0000	0.471	6.95	0.0000	0.358
1	27.1	1.07	1.46	22.6	0.987	1.38	19.6	0.868	1.26
2	55.1	2.15	2.42	44.8	2.06	2.34	37.7	1.94	2.21
3	92.7	3.18	3.40	74.3	3.10	3.31	61.2	2.98	3.19
4	140	4.20	4.38	111	4.12	4.29	90.2	3.99	4.17
5	196	5.22	5.37	155	5.13	5.28	125	5.01	5.15
6	263	6.23	6.36	206	6.14	6.27	165	6.02	6.14
7	338	7.23	7.35	264	7.15	7.26	210	7.02	7.14
8	424	8.24	8.34	330	8.16	8.26	261	8.03	8.13
9	519	9.25	9.34	403	9.16	9.25	317	9.03	9.13
10	623	10.2	10.3	483	10.2	10.2	379	10.0	10.1
11	737	11.3	11.3	570	11.2	11.2	446	11.0	11.1
12	860	12.3	12.3	665	12.2	12.2	519	12.0	12.1
13	993	13.3	13.3	766	13.2	13.2	597	13.0	13.1
14	1140	14.3	14.3	875	14.2	14.2	681	14.0	14.1
15	1290	15.3	15.3	991	15.2	15.2	770	15.0	15.1
16	1450	16.3	16.3	1110	16.2	16.2	865	16.0	16.1
17	1620	17.3	17.3	1250	17.2	17.2	965	17.1	17.1
18	1800	18.3	18.3	1380	18.2	18.2	1070	18.1	18.1
19	1990	19.3	19.3	1530	19.2	19.2	1180	19.1	19.1

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.70.

	S= 2.0			S= 3.0			S= 5.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	8.57	ACCEPT							
0	9.97	0.0000	0.0167						
1	19.3	0.569	0.846	29.0	ACCEPT				
2	33.1	1.55	1.79	39.4	1.22	1.37			
3	50.4	2.57	2.76	54.9	2.17	2.34			
4	71.1	3.58	3.74	73.6	3.17	3.32	123	ACCEPT	
5	95.3	4.59	4.73	95.1	4.18	4.30	135	3.40	3.45
6	123	5.60	5.72	119	5.18	5.29	157	4.36	4.44
7	154	6.60	6.71	146	6.18	6.28	182	5.35	5.43
8	188	7.61	7.70	176	7.19	7.28	209	6.35	6.42
9	226	8.61	8.70	209	8.19	8.27	240	7.35	7.42
10	267	9.61	9.69	244	9.19	9.27	272	8.35	8.41
11	311	10.6	10.7	282	10.2	10.3	307	9.35	9.41
12	359	11.6	11.7	323	11.2	11.3	345	10.3	10.4
13	411	12.6	12.7	366	12.2	12.3	384	11.3	11.4
14	465	13.6	13.7	412	13.2	13.3	426	12.3	12.4
15	523	14.6	14.7	461	14.2	14.3	470	13.3	13.4
16	585	15.6	15.7	512	15.2	15.3	517	14.3	14.4
17	650	16.6	16.7	567	16.2	16.2	565	15.3	15.4
18	718	17.6	17.7	624	17.2	17.2	616	16.3	16.4
19	790	18.6	18.7	683	18.2	18.2	669	17.3	17.4

$\bar{\lambda} = 0.80.$

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.80.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.0321	ACCEPT		0.0658	ACCEPT		0.101	ACCEPT	
0	26.0	0.0000	0.724	14.8	0.0000	0.699	11.1	0.0000	0.674
1	93.7	1.25	1.65	50.9	1.23	1.62	36.7	1.20	1.60
2	202	2.33	2.61	108	2.31	2.58	76.5	2.28	2.56
3	352	3.37	3.58	186	3.34	3.56	131	3.32	3.53
4	543	4.39	4.57	285	4.36	4.54	199	4.34	4.52
5	774	5.40	5.55	405	5.38	5.53	281	5.35	5.50
6	1050	6.41	6.54	546	6.39	6.52	378	6.37	6.49
7	1360	7.42	7.54	707	7.40	7.51	489	7.37	7.49
8	1720	8.43	8.53	890	8.40	8.51	615	8.38	8.48
9	2110	9.43	9.53	1090	9.41	9.50	754	9.38	9.48
10	2550	10.4	10.5	1320	10.4	10.5	908	10.4	10.5
11	3020	11.4	11.5	1560	11.4	11.5	1080	11.4	11.5
12	3540	12.4	12.5	1830	12.4	12.5	1260	12.4	12.5
13	4100	13.4	13.5	2120	13.4	13.5	1450	13.4	13.5
14	4700	14.4	14.5	2430	14.4	14.5	1670	14.4	14.5
15	5340	15.4	15.5	2750	15.4	15.5	1890	15.4	15.5
16	6030	16.5	16.5	3110	16.4	16.5	2130	16.4	16.5
17	6750	17.5	17.5	3480	17.4	17.5	2380	17.4	17.5
18	7510	18.5	18.5	3870	18.4	18.5	2650	18.4	18.5
19	8320	19.5	19.5	4280	19.4	19.5	2930	19.4	19.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.80.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.179	ACCEPT		0.265	ACCEPT		0.417	ACCEPT	
0	8.16	0.0000	0.626	6.94	0.0000	0.579	6.09	0.0000	0.510
1	25.4	1.16	1.55	20.6	1.11	1.50	17.2	1.04	1.43
2	51.5	2.23	2.51	40.8	2.19	2.46	33.0	2.11	2.38
3	86.4	3.27	3.48	67.6	3.22	3.43	53.7	3.15	3.36
4	130	4.29	4.47	101	4.24	4.42	79.1	4.17	4.34
5	183	5.31	5.45	141	5.26	5.40	109	5.18	5.33
6	244	6.32	6.45	187	6.27	6.40	144	6.19	6.32
7	315	7.32	7.44	240	7.27	7.39	184	7.20	7.31
8	394	8.33	8.43	300	8.28	8.38	229	8.21	8.31
9	482	9.33	9.43	366	9.28	9.38	278	9.21	9.30
10	579	10.3	10.4	438	10.3	10.4	332	10.2	10.3
11	685	11.3	11.4	517	11.3	11.4	391	11.2	11.3
12	800	12.3	12.4	603	12.3	12.4	455	12.2	12.3
13	924	13.3	13.4	695	13.3	13.4	524	13.2	13.3
14	1060	14.3	14.4	794	14.3	14.4	597	14.2	14.3
15	1200	15.3	15.4	899	15.3	15.4	675	15.2	15.3
16	1350	16.4	16.4	1010	16.3	16.4	758	16.2	16.3
17	1510	17.4	17.4	1130	17.3	17.4	846	17.2	17.3
18	1670	18.4	18.4	1250	18.3	18.4	939	18.2	18.3
19	1850	19.4	19.4	1390	19.3	19.4	1040	19.2	19.3

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.80.

	S= 2.0			S= 3.0			S= 5.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	1.25	ACCEPT		3.75	ACCEPT				
0	5.62	0.0000	0.292	6.92	0.0000	0.0920			
1	14.0	0.814	1.18	14.3	0.627	0.942	21.9	ACCEPT	
2	25.0	1.88	2.14	23.9	1.65	1.89	29.0	1.26	1.40
3	38.7	2.91	3.11	35.6	2.67	2.86	39.1	2.21	2.37
4	55.2	3.93	4.09	49.3	3.69	3.85	51.0	3.21	3.35
5	74.3	4.94	5.08	65.0	4.70	4.83	64.3	4.22	4.33
6	96.2	5.95	6.07	82.7	5.70	5.82	79.2	5.22	5.32
7	121	6.95	7.06	102	6.71	6.82	95.4	6.22	6.32
8	148	7.96	8.06	124	7.71	7.81	113	7.22	7.31
9	178	8.96	9.05	148	8.72	8.80	132	8.23	8.31
10	211	9.97	10.0	173	9.72	9.80	153	9.23	9.30
11	246	11.0	11.0	201	10.7	10.8	175	10.2	10.3
12	284	12.0	12.0	231	11.7	11.8	198	11.2	11.3
13	325	13.0	13.0	263	12.7	12.8	223	12.2	12.3
14	369	14.0	14.0	296	13.7	13.8	249	13.2	13.3
15	415	15.0	15.0	332	14.7	14.8	277	14.2	14.3
16	464	16.0	16.0	370	15.7	15.8	306	15.2	15.3
17	516	17.0	17.0	409	16.7	16.8	337	16.2	16.3
18	570	18.0	18.0	451	17.7	17.8	369	17.2	17.3
19	627	19.0	19.0	495	18.7	18.8	402	18.2	18.3

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.80.

	S= 7.0			S=10.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE	
	53.3	ACCEPT				
4	63.3	2.76	2.85			
5	75.5	3.74	3.84			
6	89.3	4.74	4.83	138	ACCEPT	
7	104	5.74	5.82	143	5.05	5.07
8	121	6.74	6.81	157	6.01	6.06
9	139	7.74	7.81	174	7.00	7.06
10	157	8.74	8.80	192	8.00	8.05
11	178	9.74	9.80	211	9.00	9.05
12	199	10.7	10.8	231	10.0	10.0
13	221	11.7	11.8	252	11.0	11.0
14	245	12.7	12.8	275	12.0	12.0
15	270	13.7	13.8	298	13.0	13.0
16	296	14.7	14.8	323	14.0	14.0
17	323	15.7	15.8	348	15.0	15.0
18	351	16.7	16.8	375	16.0	16.0
19	381	17.7	17.8	402	17.0	17.0

$\bar{\lambda} = 0.90.$

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.90.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.0125	ACCEPT		0.0253	ACCEPT		0.0383	ACCEPT	
0	25.8	0.0000	0.736	14.6	0.0000	0.723	10.8	0.0000	0.711
1	92.6	1.27	1.66	49.8	1.26	1.65	35.6	1.25	1.64
2	200	2.34	2.62	106	2.33	2.61	74.0	2.32	2.60
3	348	3.38	3.60	182	3.37	3.59	126	3.36	3.57
4	536	4.40	4.58	278	4.39	4.57	192	4.38	4.56
5	765	5.42	5.57	395	5.41	5.56	272	5.40	5.55
6	1030	6.43	6.56	532	6.42	6.55	365	6.41	6.54
7	1340	7.44	7.55	690	7.42	7.54	472	7.41	7.53
8	1690	8.44	8.54	868	8.43	8.53	593	8.42	8.52
9	2080	9.45	9.54	1070	9.44	9.53	728	9.42	9.52
10	2510	10.5	10.5	1290	10.4	10.5	876	10.4	10.5
11	2990	11.5	11.5	1530	11.4	11.5	1040	11.4	11.5
12	3500	12.5	12.5	1790	12.4	12.5	1210	12.4	12.5
13	4050	13.5	13.5	2070	13.4	13.5	1400	13.4	13.5
14	4640	14.5	14.5	2370	14.4	14.5	1610	14.4	14.5
15	5270	15.5	15.5	2690	15.5	15.5	1820	15.4	15.5
16	5950	16.5	16.5	3030	16.5	16.5	2050	16.4	16.5
17	6660	17.5	17.5	3390	17.5	17.5	2300	17.4	17.5
18	7420	18.5	18.5	3770	18.5	18.5	2560	18.4	18.5
19	8210	19.5	19.5	4180	19.5	19.5	2830	19.4	19.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.90.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.0654	ACCEPT		0.0937	ACCEPT		0.139	ACCEPT	
0	7.86	0.0000	0.687	6.59	0.0000	0.664	5.65	0.0000	0.631
1	24.2	1.23	1.61	19.3	1.21	1.59	15.7	1.18	1.56
2	48.9	2.30	2.58	38.1	2.28	2.55	30.1	2.25	2.52
3	81.9	3.34	3.55	63.0	3.32	3.53	48.8	3.29	3.49
4	123	4.36	4.53	93.9	4.34	4.51	71.8	4.31	4.48
5	173	5.37	5.52	131	5.35	5.50	99.2	5.32	5.47
6	231	6.38	6.51	174	6.36	6.49	131	6.33	6.46
7	298	7.39	7.51	223	7.37	7.48	167	7.34	7.45
8	373	8.40	8.50	278	8.38	8.48	207	8.34	8.44
9	456	9.40	9.50	340	9.38	9.47	252	9.35	9.44
10	548	10.4	10.5	407	10.4	10.5	301	10.4	10.4
11	648	11.4	11.5	480	11.4	11.5	354	11.4	11.4
12	756	12.4	12.5	560	12.4	12.5	412	12.4	12.4
13	873	13.4	13.5	645	13.4	13.5	474	13.4	13.4
14	998	14.4	14.5	737	14.4	14.5	541	14.4	14.4
15	1130	15.4	15.5	835	15.4	15.5	611	15.4	15.4
16	1270	16.4	16.5	939	16.4	16.5	686	16.4	16.4
17	1420	17.4	17.5	1050	17.4	17.5	766	17.4	17.4
18	1580	18.4	18.5	1160	18.4	18.5	850	18.4	18.4
19	1750	19.4	19.5	1290	19.4	19.4	938	19.4	19.4

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.90.

	S= 2.0			S= 3.0			S= 5.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	0.318	ACCEPT		0.556	ACCEPT		1.39	ACCEPT	
0	4.64	0.0000	0.527	4.44	0.0000	0.428	4.76	0.0000	0.240
1	11.7	1.07	1.45	10.6	0.973	1.34	10.3	0.779	1.12
2	21.0	2.15	2.41	18.2	2.04	2.30	16.8	1.84	2.07
3	32.5	3.18	3.38	27.5	3.08	3.27	24.4	2.87	3.05
4	46.4	4.20	4.37	38.3	4.09	4.26	32.9	3.88	4.03
5	62.5	5.21	5.35	50.8	5.11	5.24	42.5	4.89	5.02
6	81.0	6.22	6.35	64.8	6.11	6.23	53.1	5.90	6.01
7	102	7.23	7.34	80.4	7.12	7.23	64.7	6.90	7.01
8	125	8.23	8.33	97.6	8.13	8.22	77.3	7.91	8.00
9	150	9.24	9.33	116	9.13	9.22	91.0	8.91	9.00
10	178	10.2	10.3	137	10.1	10.2	106	9.91	9.99
11	207	11.2	11.3	159	11.1	11.2	121	10.9	11.0
12	240	12.2	12.3	182	12.1	12.2	138	11.9	12.0
13	274	13.2	13.3	207	13.1	13.2	156	12.9	13.0
14	311	14.3	14.3	234	14.1	14.2	175	13.9	14.0
15	350	15.3	15.3	263	15.1	15.2	194	14.9	15.0
16	391	16.3	16.3	293	16.1	16.2	215	15.9	16.0
17	435	17.3	17.3	324	17.1	17.2	237	16.9	17.0
18	480	18.3	18.3	357	18.1	18.2	260	17.9	18.0
19	529	19.3	19.3	392	19.1	19.2	284	18.9	19.0

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 0.90.

	S= 7.0			S=10.0			S=20.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
	3.89	ACCEPT							
0	6.12	0.0000	0.0660	13.1	ACCEPT				
1	11.2	0.630	0.906	14.6	0.534	0.577			
2	17.3	1.64	1.86	20.0	1.36	1.52			
3	24.2	2.66	2.83	26.4	2.35	2.50			
4	31.9	3.67	3.81	33.6	3.35	3.48			
5	40.3	4.68	4.80	41.3	4.36	4.47	73.7	ACCEPT	
6	49.6	5.68	5.79	49.6	5.36	5.46	74.3	4.34	4.34
7	59.6	6.69	6.78	58.6	6.36	6.45	81.5	5.29	5.34
8	70.3	7.69	7.78	68.1	7.36	7.44	89.4	6.28	6.33
9	81.9	8.69	8.77	78.2	8.37	8.44	98.0	7.28	7.33
10	94.2	9.70	9.77	88.9	9.37	9.44	107	8.28	8.33
11	107	10.7	10.8	100	10.4	10.4	116	9.28	9.32
12	121	11.7	11.8	112	11.4	11.4	126	10.3	10.3
13	136	12.7	12.8	125	12.4	12.4	137	11.3	11.3
14	151	13.7	13.8	138	13.4	13.4	148	12.3	12.3
15	168	14.7	14.8	151	14.4	14.4	159	13.3	13.3
16	185	15.7	15.8	165	15.4	15.4	170	14.3	14.3
17	202	16.7	16.8	180	16.4	16.4	182	15.3	15.3
18	221	17.7	17.8	196	17.4	17.4	194	16.3	16.3
19	240	18.7	18.8	212	18.4	18.4	207	17.3	17.3

$\bar{\lambda} = 1.00.$

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 1.00.

	S= 0.1			S= 0.2			S= 0.3		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	25.6	0.0000	0.746	14.4	0.0000	0.743	10.6	0.0000	0.741
1	91.8	1.28	1.67	49.0	1.28	1.67	34.7	1.28	1.67
2	198	2.35	2.63	104	2.35	2.63	72.1	2.36	2.63
3	344	3.39	3.61	178	3.39	3.61	123	3.39	3.61
4	531	4.41	4.59	273	4.41	4.59	187	4.41	4.59
5	757	5.43	5.58	388	5.43	5.58	264	5.43	5.58
6	1020	6.44	6.57	522	6.44	6.57	355	6.44	6.57
7	1330	7.45	7.56	677	7.45	7.56	459	7.45	7.56
8	1680	8.45	8.56	852	8.45	8.56	577	8.45	8.56
9	2060	9.46	9.55	1050	9.46	9.55	708	9.46	9.55
10	2490	10.5	10.5	1260	10.5	10.5	852	10.5	10.5
11	2960	11.5	11.5	1500	11.5	11.5	1010	11.5	11.5
12	3460	12.5	12.5	1750	12.5	12.5	1180	12.5	12.5
13	4010	13.5	13.5	2020	13.5	13.5	1360	13.5	13.5
14	4590	14.5	14.5	2320	14.5	14.5	1560	14.5	14.5
15	5220	15.5	15.5	2630	15.5	15.5	1770	15.5	15.5
16	5890	16.5	16.5	2970	16.5	16.5	2000	16.5	16.5
17	6590	17.5	17.5	3320	17.5	17.5	2230	17.5	17.5
18	7340	18.5	18.5	3700	18.5	18.5	2480	18.5	18.5
19	8130	19.5	19.5	4090	19.5	19.5	2750	19.5	19.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 1.00.

	S= 0.5			S= 0.7			S= 1.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	7.65	0.0000	0.736	6.36	0.0000	0.733	5.40	0.0000	0.730
1	23.3	1.28	1.67	18.4	1.28	1.66	14.7	1.29	1.66
2	47.0	2.36	2.63	36.2	2.36	2.63	28.1	2.36	2.63
3	78.6	3.39	3.61	59.6	3.39	3.60	45.4	3.40	3.60
4	118	4.42	4.59	88.8	4.42	4.59	66.7	4.42	4.59
5	166	5.43	5.58	124	5.43	5.58	92.1	5.43	5.58
6	222	6.44	6.57	164	6.44	6.57	121	6.44	6.57
7	285	7.45	7.56	211	7.45	7.56	155	7.45	7.56
8	357	8.45	8.56	263	8.45	8.55	192	8.45	8.55
9	437	9.46	9.55	320	9.46	9.55	233	9.46	9.55
10	524	10.5	10.5	384	10.5	10.5	279	10.5	10.5
11	620	11.5	11.5	453	11.5	11.5	328	11.5	11.5
12	724	12.5	12.5	528	12.5	12.5	381	12.5	12.5
13	835	13.5	13.5	609	13.5	13.5	439	13.5	13.5
14	955	14.5	14.5	695	14.5	14.5	500	14.5	14.5
15	1080	15.5	15.5	787	15.5	15.5	565	15.5	15.5
16	1220	16.5	16.5	885	16.5	16.5	635	16.5	16.5
17	1360	17.5	17.5	988	17.5	17.5	708	17.5	17.5
18	1510	18.5	18.5	1100	18.5	18.5	785	18.5	18.5
19	1670	19.5	19.5	1210	19.5	19.5	867	19.5	19.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 1.00.

	S= 2.0			S= 3.0			S= 5.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	4.28	0.0000	0.723	3.90	0.0000	0.720	3.60	0.0000	0.716
1	10.4	1.29	1.66	9.02	1.29	1.65	7.88	1.29	1.65
2	18.6	2.36	2.62	15.5	2.36	2.62	12.9	2.37	2.61
3	28.8	3.40	3.60	23.2	3.40	3.60	18.8	3.40	3.59
4	41.0	4.42	4.58	32.4	4.42	4.58	25.5	4.42	4.58
5	55.1	5.43	5.57	42.8	5.43	5.57	32.9	5.43	5.57
6	71.3	6.44	6.57	54.6	6.44	6.56	41.2	6.44	6.56
7	89.5	7.45	7.56	67.7	7.45	7.56	50.3	7.45	7.56
8	110	8.45	8.55	82.1	8.45	8.55	60.1	8.46	8.55
9	132	9.46	9.55	97.9	9.46	9.55	70.8	9.46	9.55
10	156	10.5	10.5	115	10.5	10.5	82.3	10.5	10.5
11	182	11.5	11.5	133	11.5	11.5	94.5	11.5	11.5
12	210	12.5	12.5	153	12.5	12.5	108	12.5	12.5
13	240	13.5	13.5	174	13.5	13.5	121	13.5	13.5
14	273	14.5	14.5	197	14.5	14.5	136	14.5	14.5
15	307	15.5	15.5	221	15.5	15.5	152	15.5	15.5
16	343	16.5	16.5	246	16.5	16.5	168	16.5	16.5
17	381	17.5	17.5	272	17.5	17.5	185	17.5	17.5
18	421	18.5	18.5	300	18.5	18.5	203	18.5	18.5
19	463	19.5	19.5	329	19.5	19.5	221	19.5	19.5

SINGLE SAMPLING TABLES FOR GAMMA PRIOR WITH MEAN 1.00.

	S= 7.0			S=10.0			S=20.0		
C	M	SAMPLE SIZE		M	SAMPLE SIZE		M	SAMPLE SIZE	
0	3.47	0.0000	0.714	3.37	0.0000	0.713	3.26	0.0000	0.711
1	7.39	1.29	1.65	7.02	1.30	1.64	6.59	1.30	1.64
2	11.9	2.37	2.61	11.1	2.37	2.61	10.1	2.37	2.61
3	16.9	3.40	3.59	15.5	3.40	3.59	13.8	3.41	3.59
4	22.5	4.42	4.58	20.3	4.42	4.58	17.7	4.43	4.57
5	28.7	5.44	5.57	25.6	5.44	5.57	21.9	5.44	5.56
6	35.5	6.44	6.56	31.2	6.45	6.56	26.2	6.45	6.56
7	42.8	7.45	7.55	37.2	7.45	7.55	30.7	7.45	7.55
8	50.7	8.46	8.55	43.7	8.46	8.55	35.4	8.46	8.55
9	59.2	9.46	9.55	50.5	9.46	9.54	40.3	9.46	9.54
10	68.2	10.5	10.5	57.7	10.5	10.5	45.4	10.5	10.5
11	77.9	11.5	11.5	65.4	11.5	11.5	50.8	11.5	11.5
12	88.1	12.5	12.5	73.4	12.5	12.5	56.3	12.5	12.5
13	98.8	13.5	13.5	81.8	13.5	13.5	62.0	13.5	13.5
14	110	14.5	14.5	90.7	14.5	14.5	67.9	14.5	14.5
15	122	15.5	15.5	99.9	15.5	15.5	74.0	15.5	15.5
16	135	16.5	16.5	110	16.5	16.5	80.3	16.5	16.5
17	148	17.5	17.5	120	17.5	17.5	86.9	17.5	17.5
18	161	18.5	18.5	130	18.5	18.5	93.6	18.5	18.5
19	175	19.5	19.5	141	19.5	19.5	100	19.5	19.5

Indleveret til Selskabet den 29. oktober 1970.
Færdig fra trykkeriet den 15. juni 1971.

Det Kongelige Danske Videnskabernes Selskab

Matematisk-fysiske Skrifter

Mat. Fys. Skr. Dan. Vid. Selsk.

Bind **1** (kr. 141.–)

Bind **2** (kr. 180.–)

Bind 3 (kr. 211.–)

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1.	BARTLETT, J. H., and WAGNER, C. A.: The Restricted Problem of Three Bodies (II). 1965	17.–
2.	HALD, A.: Bayesian Single Sampling Attribute Plans for Discrete Prior Distributions. 1965	40.–
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4.	HALD, A., and KOUSGAARD, E.: A Table for Solving the Binomial Equation $B(c,n,p) = P$ for $c = 0(1)50$ and 15 Values of P. 1967	14.–
5.	ZELDES, N., GRILL, A., and SIMIEVIC, A.: Shell-Model Semi-Empirical Nuclear Masses (II). 1967	60.–
6.	LAUSTSEN, SVEND: Meridian Observations Made in Brorfelde (Copenhagen University Observatory) 1964–1967. Positions of 972 Stars Brighter than 11.0 vis. mag. 1968	14.–
7.	HALD, A., and THYREGOD, P.: Bayesian Single Sampling Plans Based on Linear Costs and the Poisson Distribution. 1971	52.–

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